Cosmic Flows

as a Probe of the

Large Scale Structure

of the

Universe

Coffee Break

- The Expanding Universe
- Peculiar Velocities
- The Cosmic Distance Ladder

Bulk Flows

- Pairwise velocities
- Optimal moments
- Concluding remarks

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University of Kansas

2004 Santa Fe

Cosmology Summer Workshop

July 16, 2004

The Expanding Universe

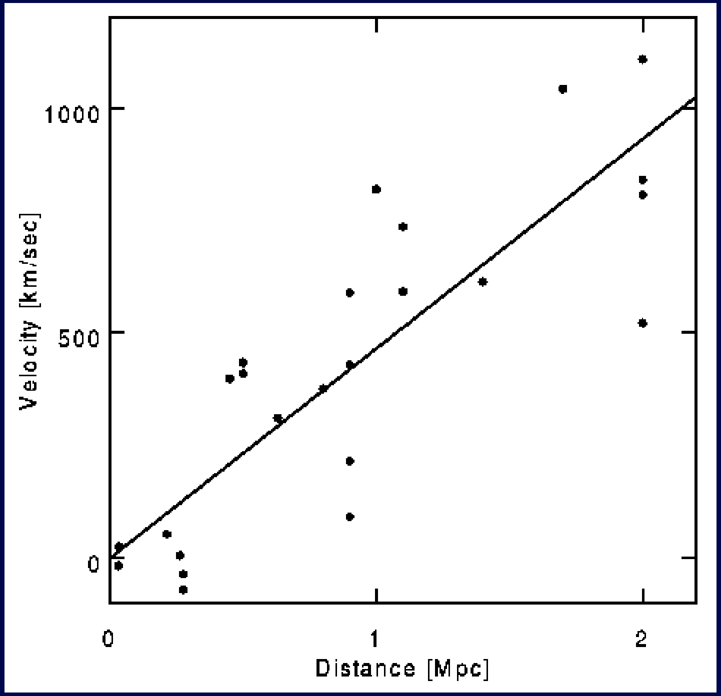
1922 – Friedmann Showed that Einstein's Equations have an expanding solution.

1929 – Hubble observed that

Distance $\propto$ Redshift

In a HOMOGENEOUS Universe: $H_o\, r = cz = c\, \delta\lambda / \lambda$

$H_o = 500$ km / s / Mpc



Hubble Original data

The Expanding Universe

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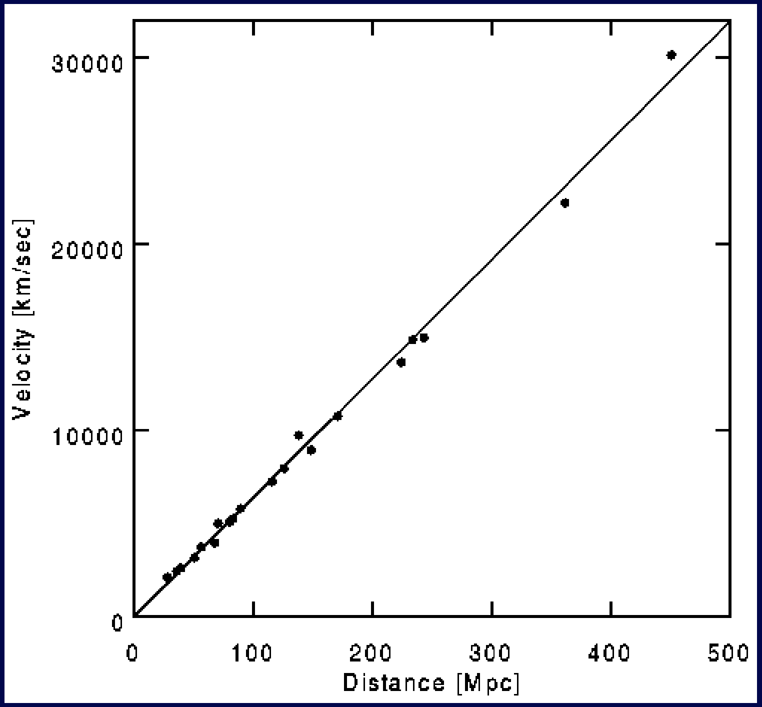
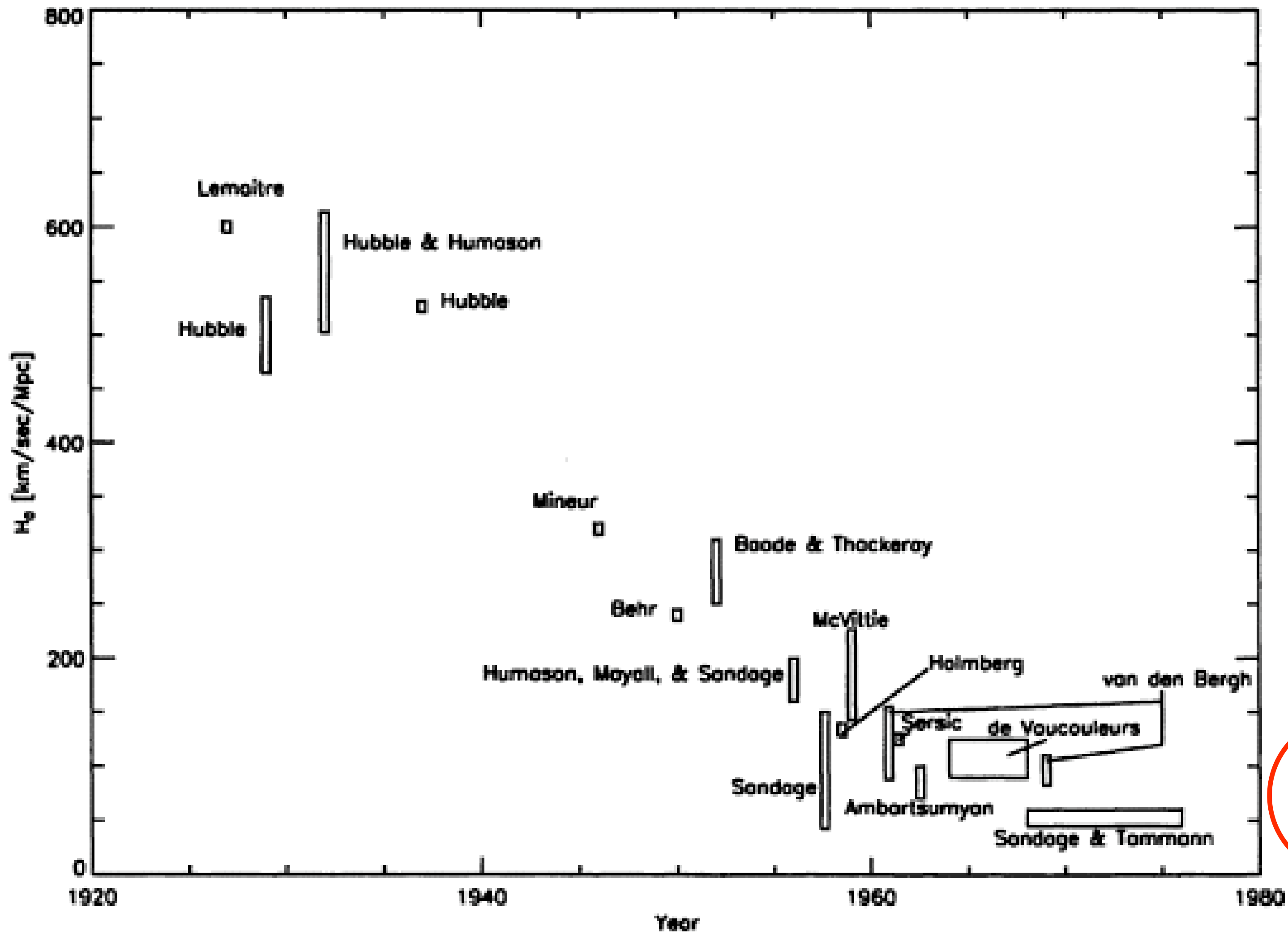
Distance ∝ Redshift

In a HOMOGENEOUS Universe: $H_o\, r = cz = c\, \delta\lambda / \lambda$



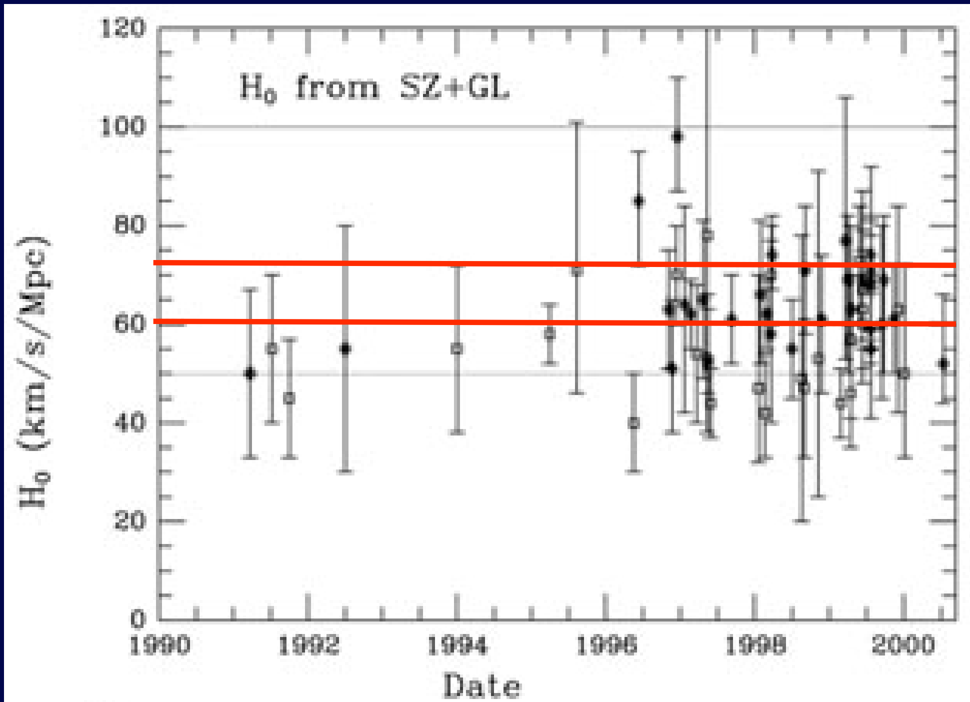
$H_o = 65 \pm 15$ km / s / Mpc

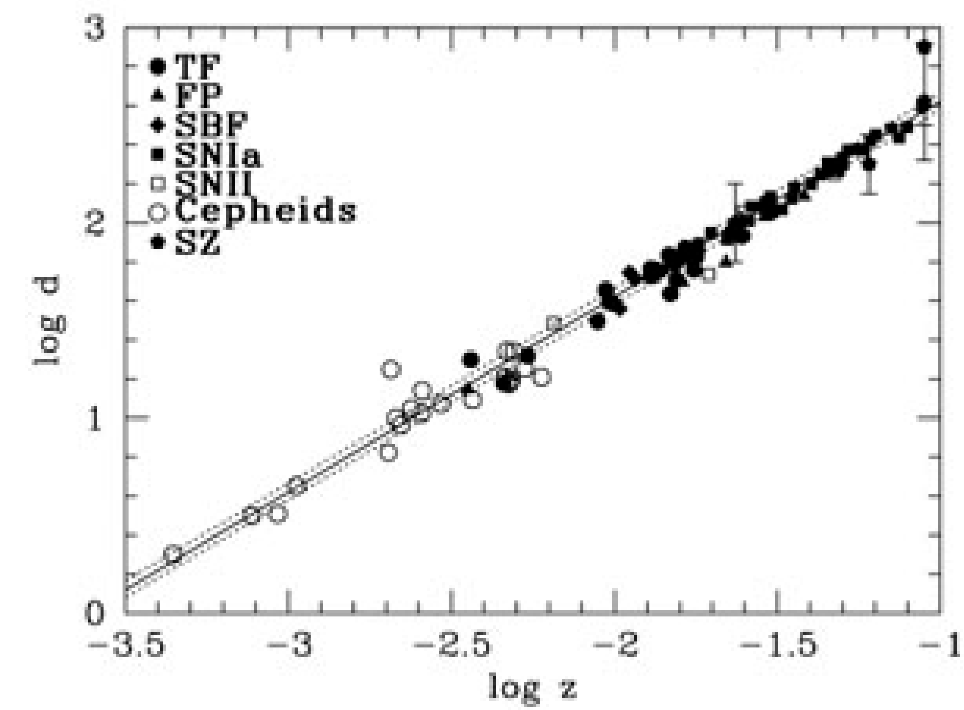
TRIMBLE



800
600
400
200
0
1920
1940
1960
1980
Year
H0 [km/sec/Mpc]
Lemaitre
Hubble & Humason
Hubble
Hubble
Mineur
Baade & Thackeray
Behr
McVittie
Humason, Mayall, & Sandage
Holmberg
van den Bergh
Sersic
de Vaucouleurs
Sandage
Ambartsumyan
Sandage & Tammann

The Hubble Constant

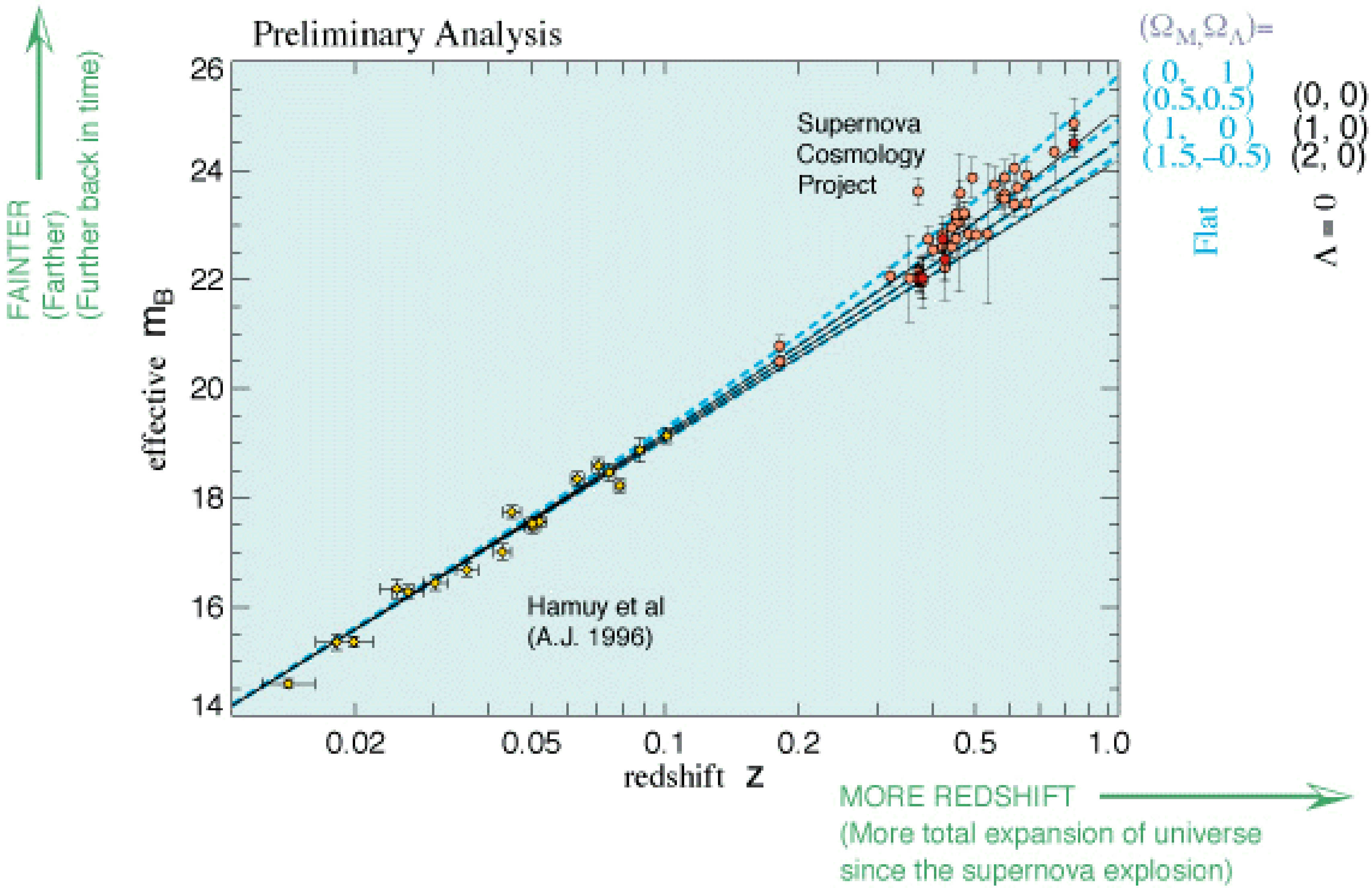




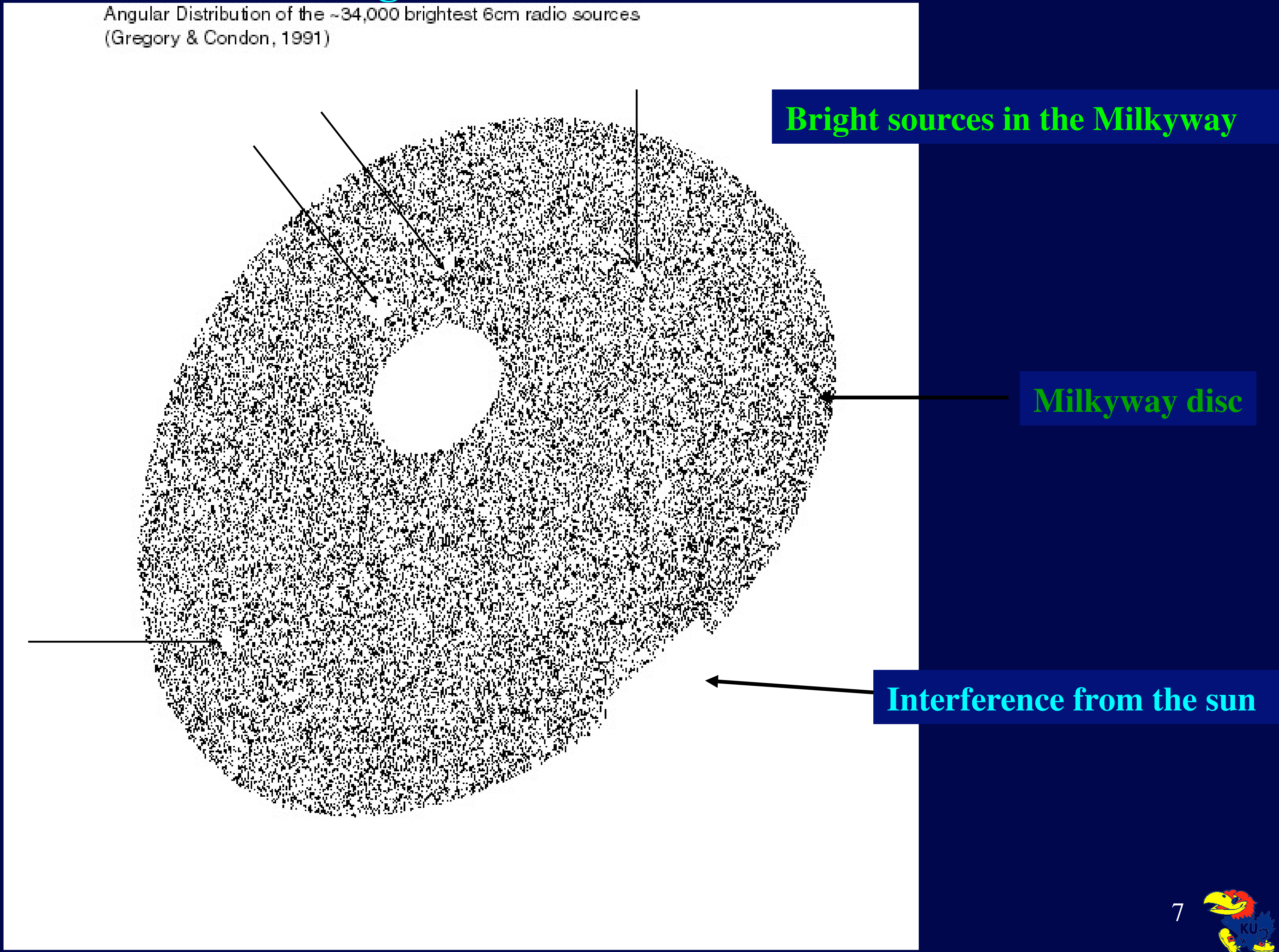
The solid line is for H0 = 72 km/s/Mpc
with the dashed lines representing ±10%

Freedman etal, 2003

Hubble Plots



Is the Universe Homogeneous ?



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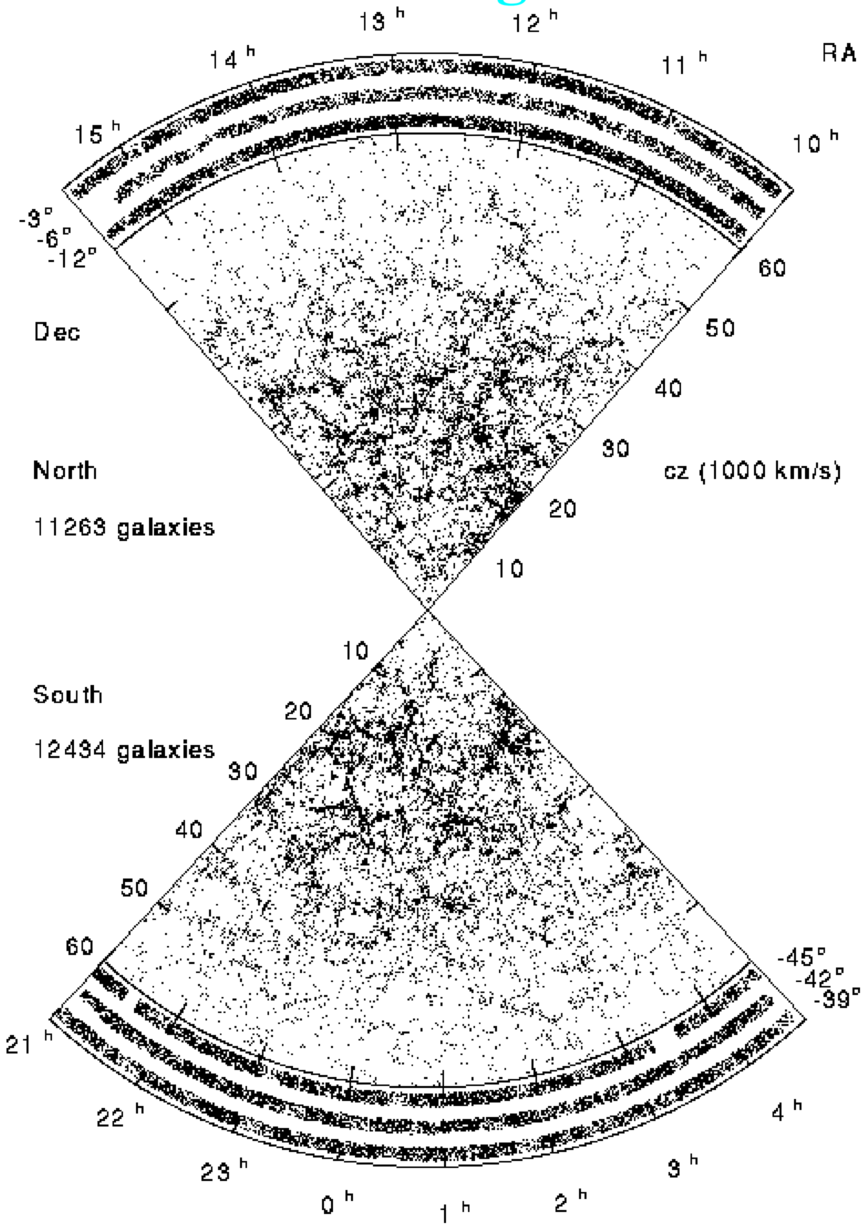

LCRS, Sheckman etal, 1996

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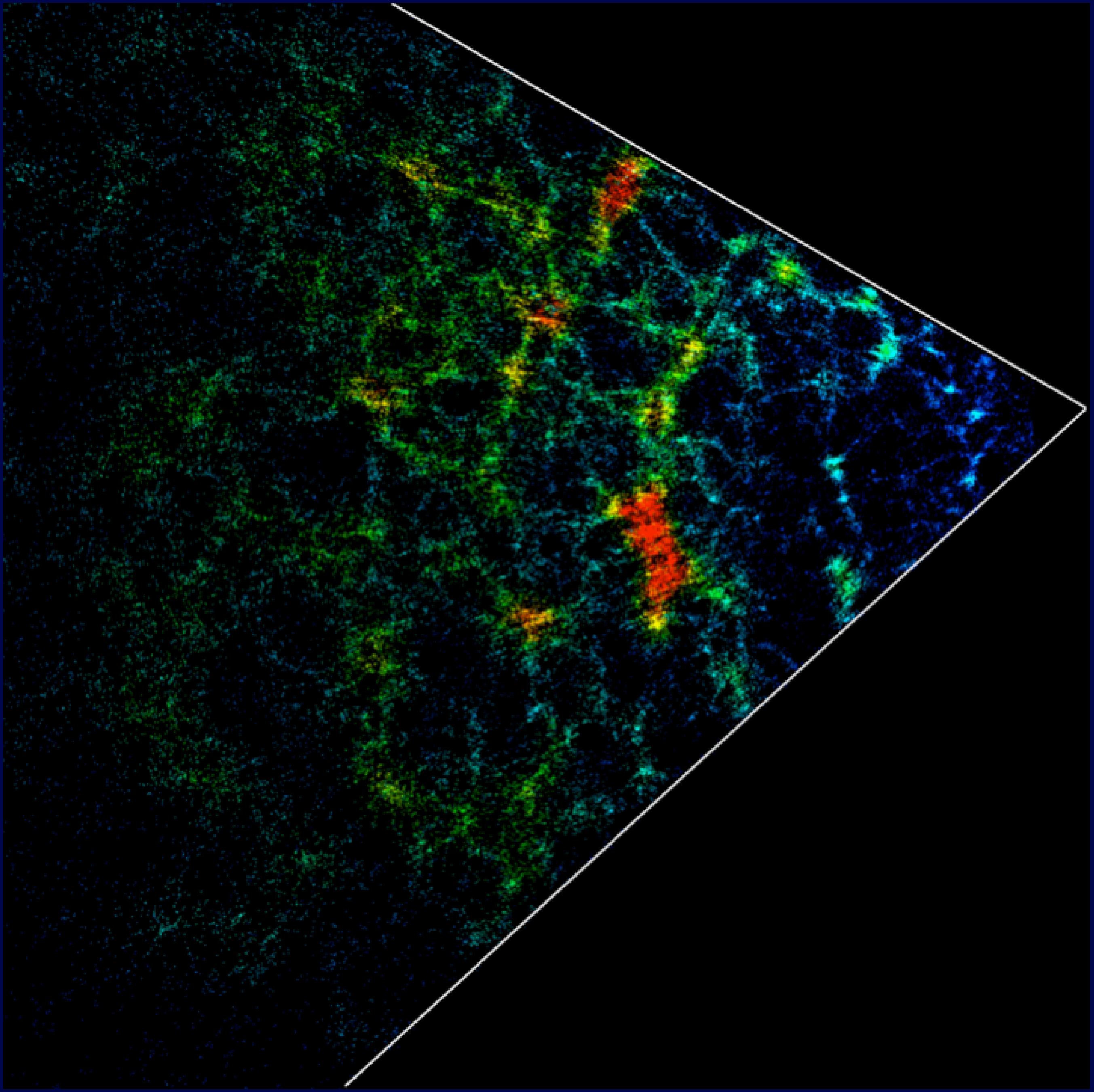
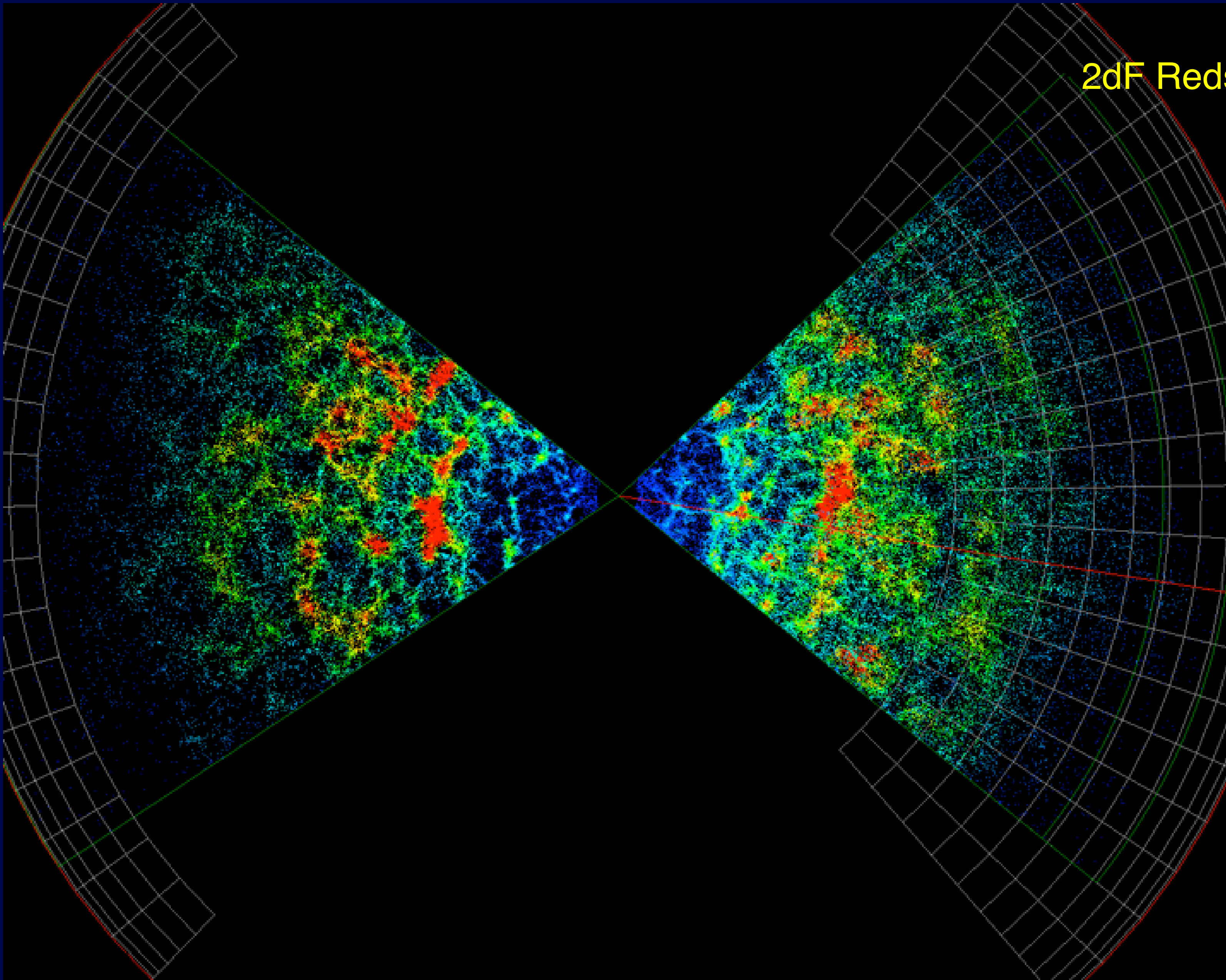
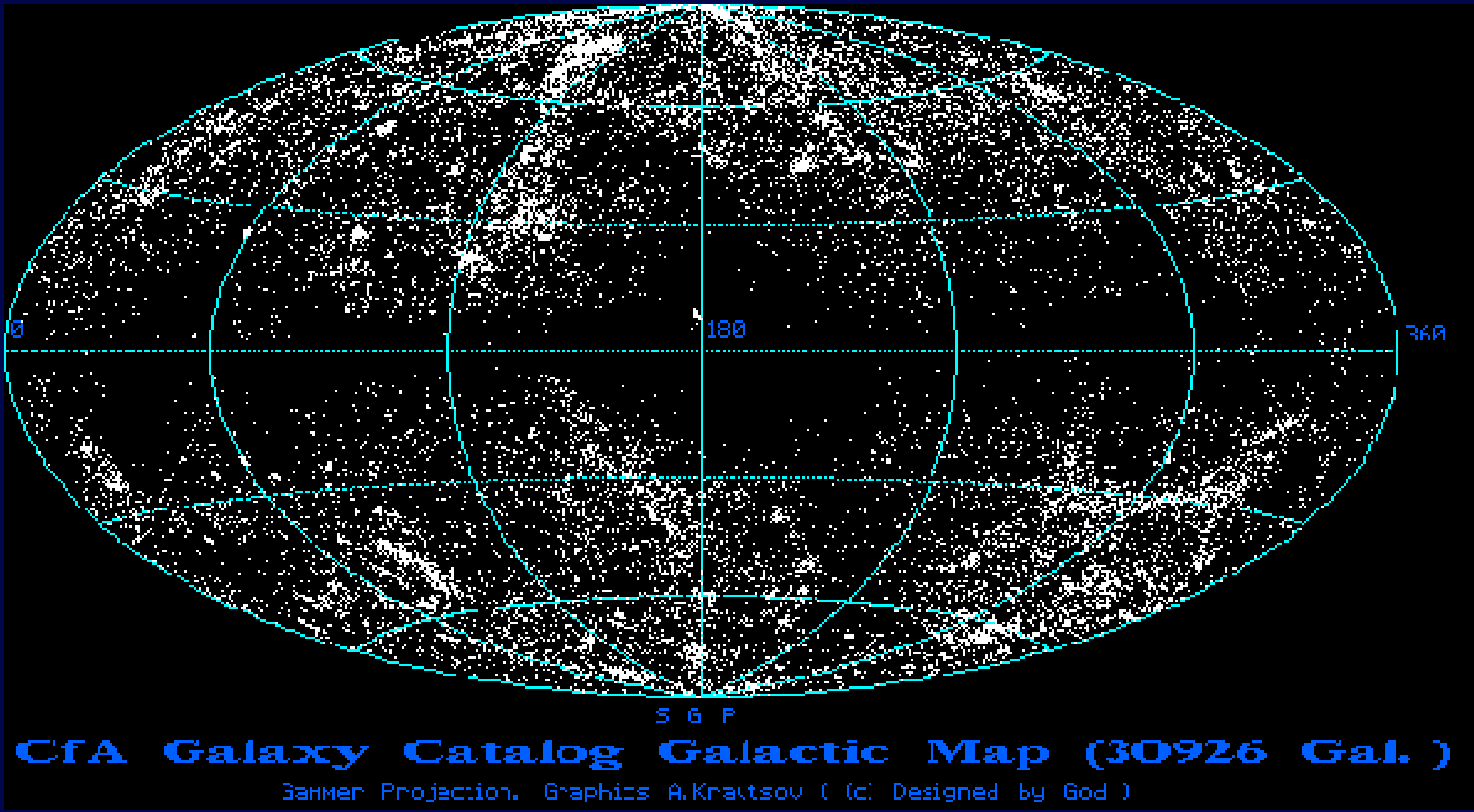


2dF Redshift Survey

Is the Universe Homogeneous ?



Is the Universe Homogeneous ?



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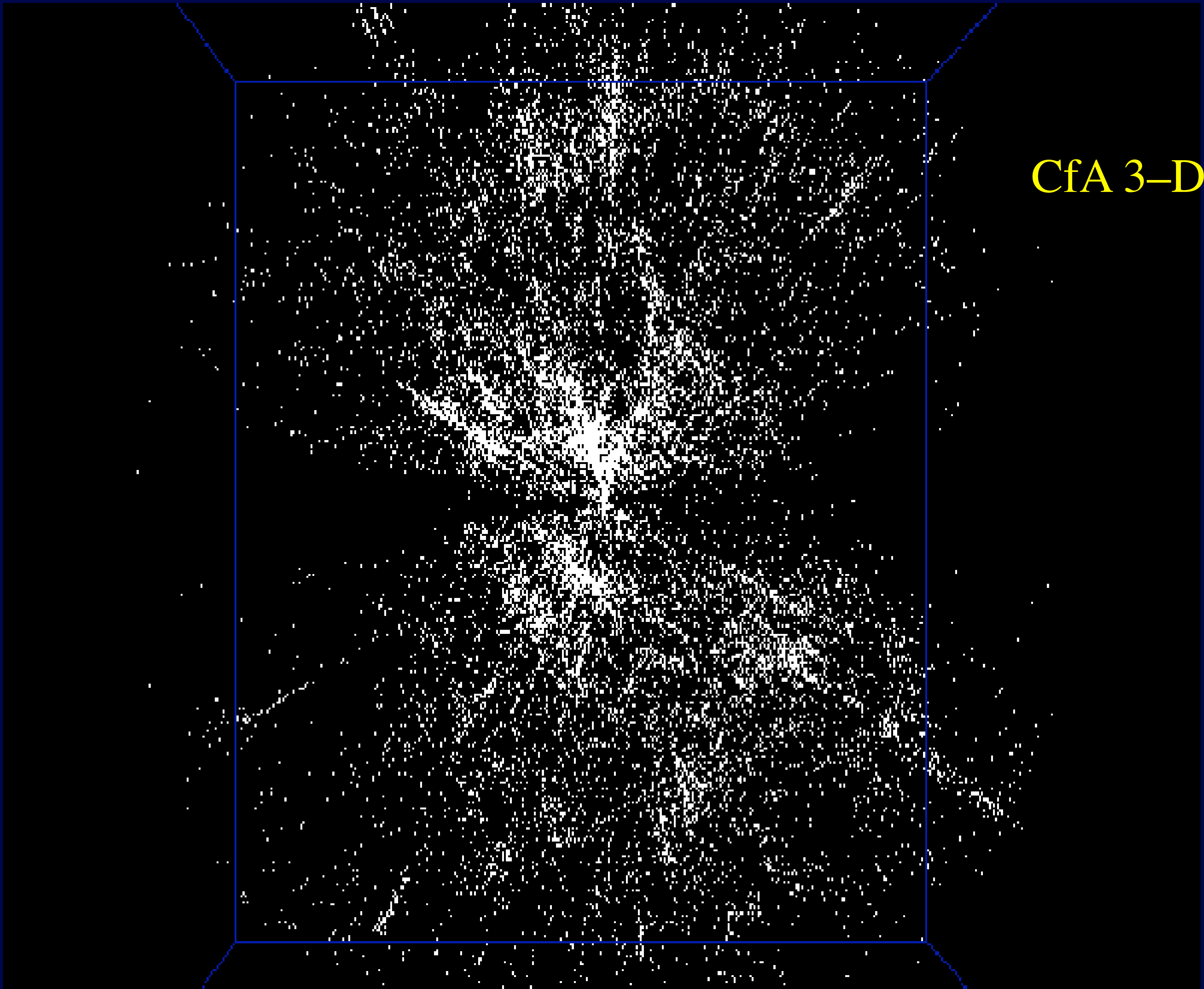
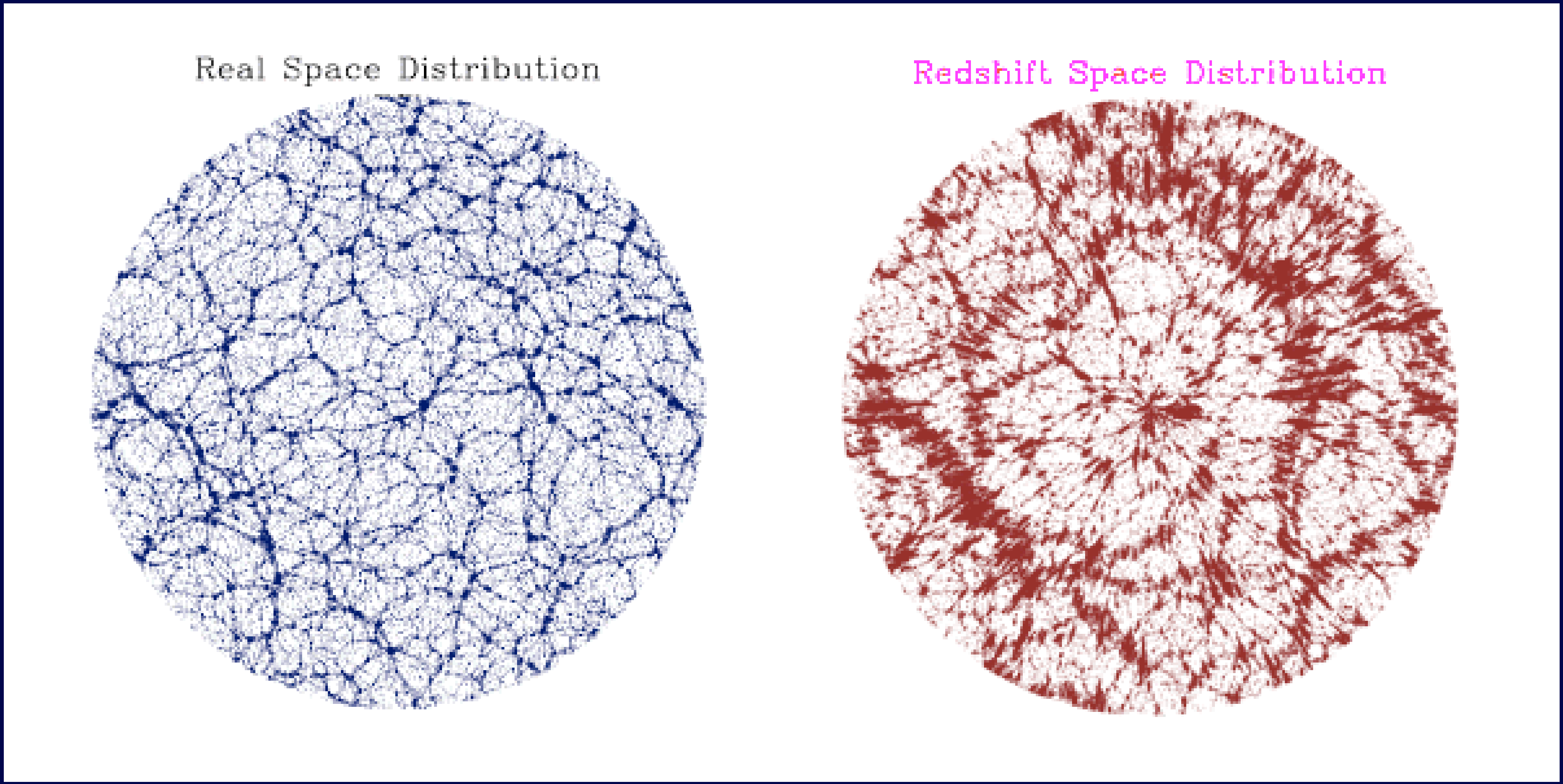
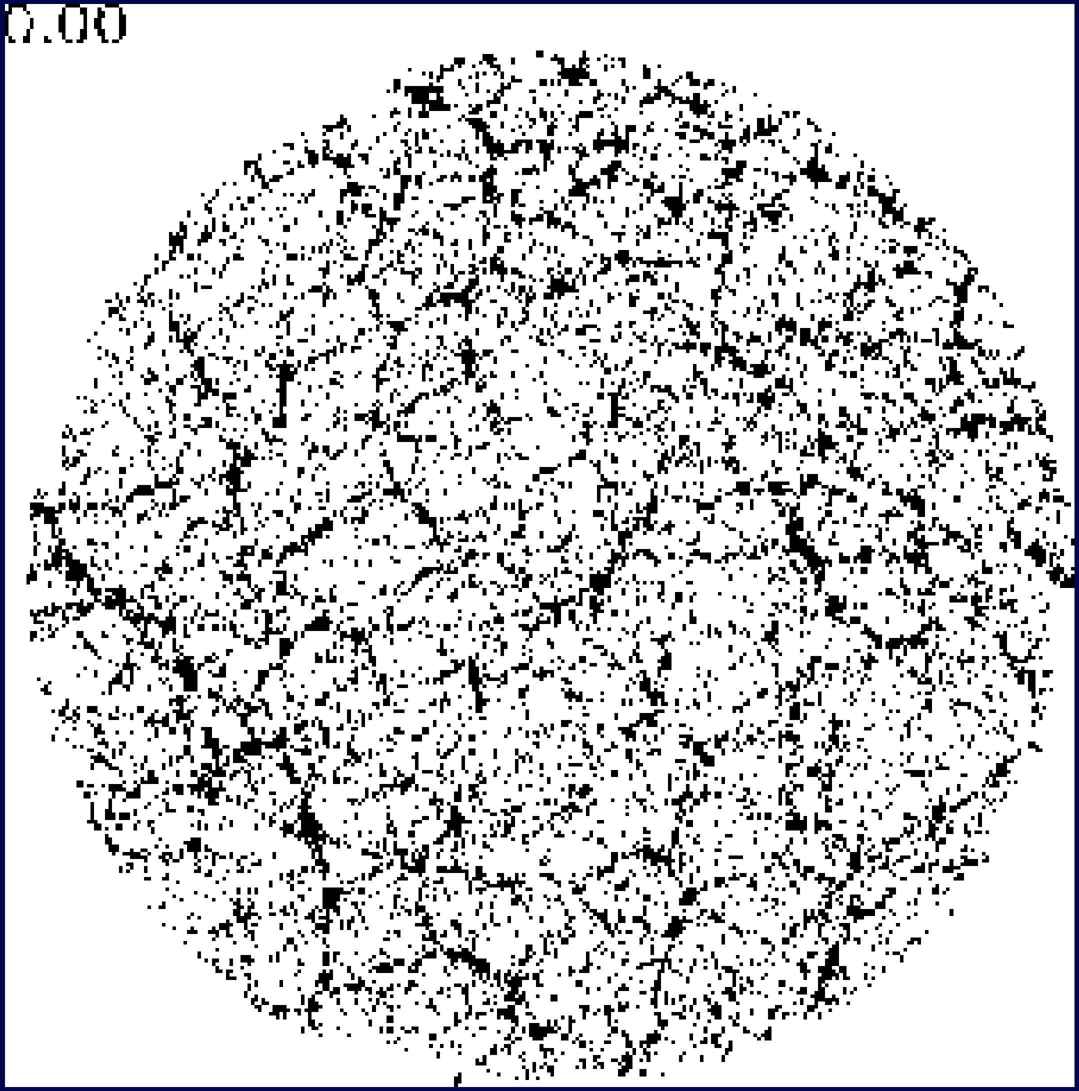


CfA 3–D

Redshift Distortions



Redshift Distortions



Peculiar Velocity Field

Measure the line of sight peculiar velocities:

$$v_p = cz - H_o r$$

The difference between the redshift and the distance

Why should we study v_p ?

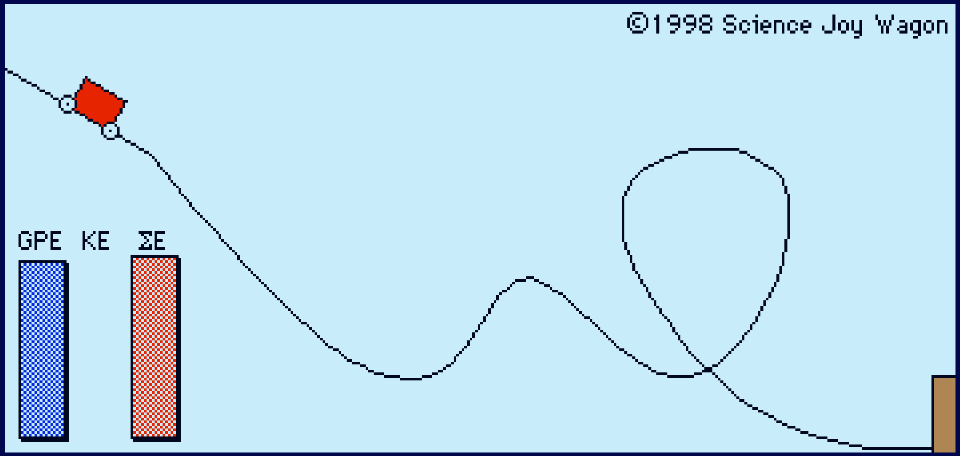
- ⋆ **The peculiar velocity field is dominated by large scales**

  ⟹ **Linear structure**

- ⋆ **Test of gravitational instability model** $\vec{\nabla} \cdot \vec{V} = \frac{\delta\rho}{\rho} \qquad \vec{\nabla} \times \vec{V} = 0$

- ⋆ **A direct probe of the mass distribution** $\vec{V} = \vec{\nabla}\phi$

- ⋆ **Comparison of velocity fields & Luminous matter distribution** ⟹ **bias, Ω ...**



- ⋆ **A direct probe of the mass distribution** $\vec{V} = \vec{\nabla}\phi$

How to measure cosmological distances?

aka The Cosmic Distance Ladder

A stepwise procedure: Errors proliferating

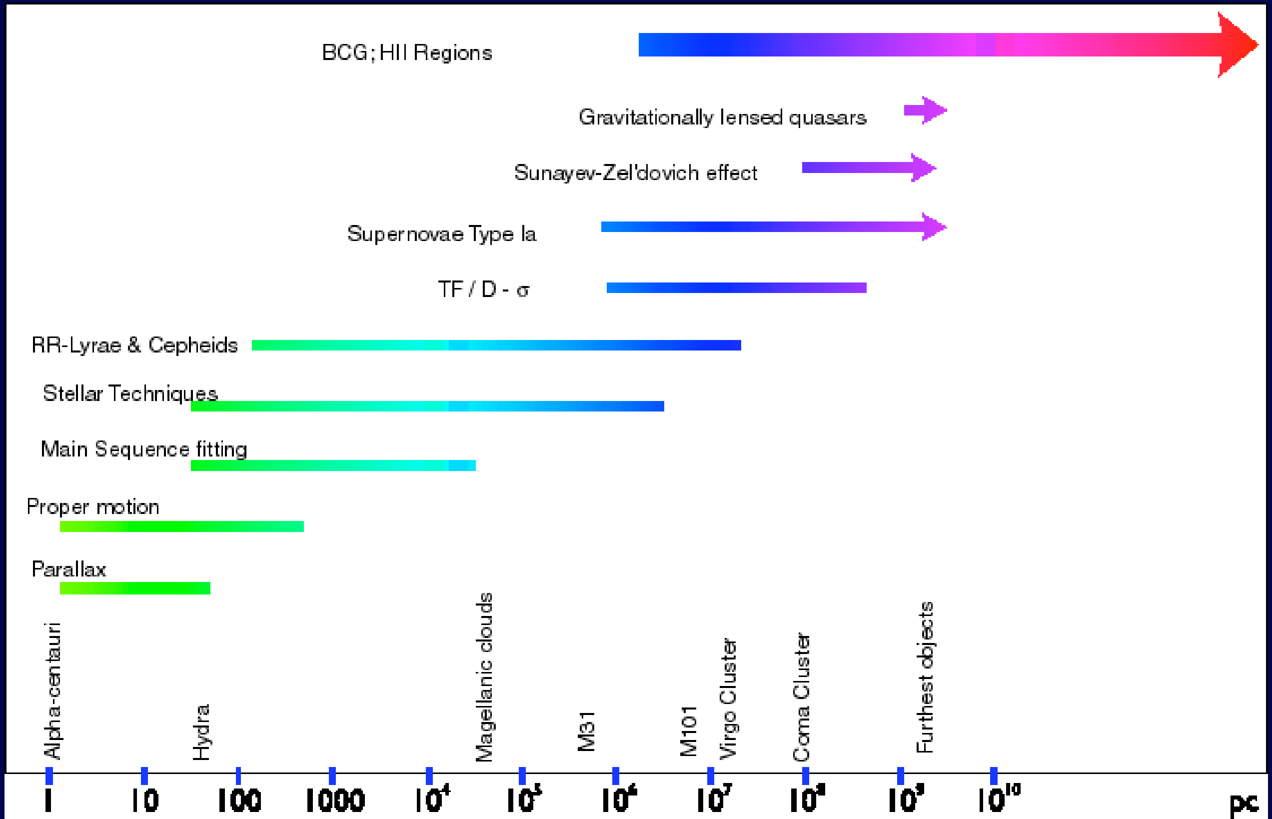
The idea:

Measure the apparent luminosity (ℓ)

Find out absolute luminosity (L)

Get the distance

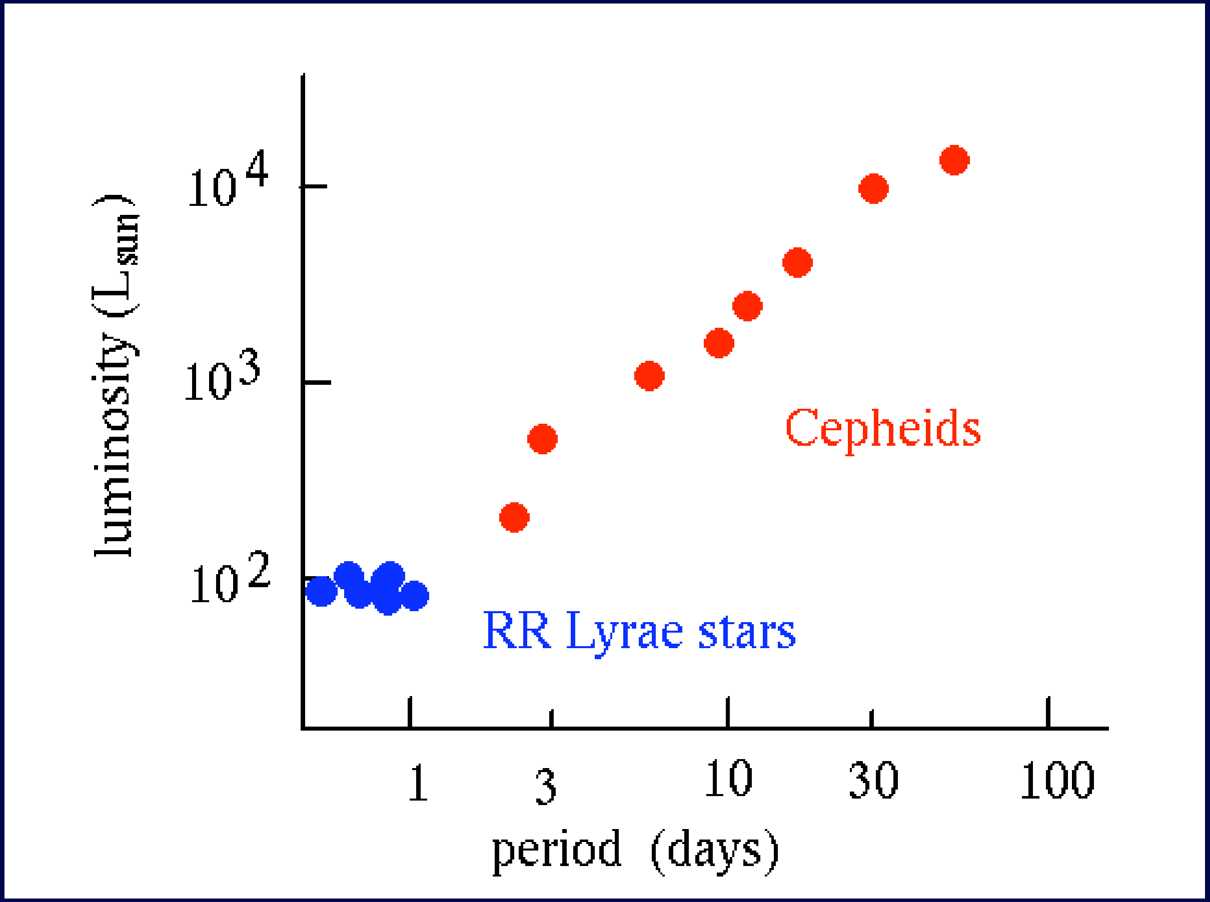
$\ell = L / 4\pi r^2$



The Cosmic Ladder

- **Find correlated observables:**

Period – Luminosity **variable stars (Cepheids, RR-Lyr, …)**



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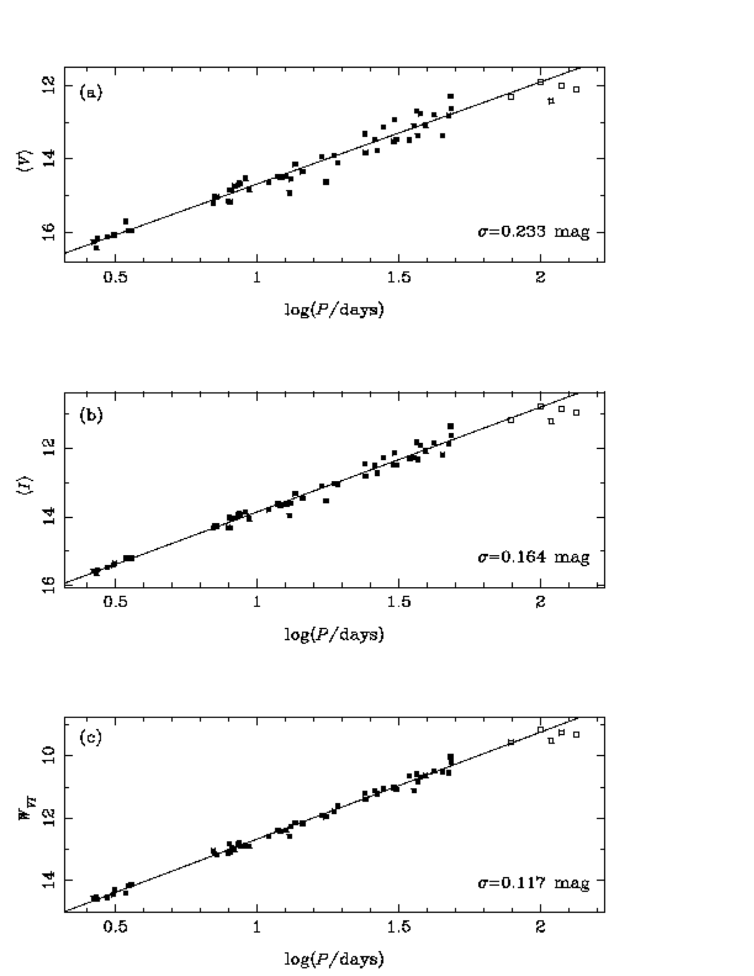

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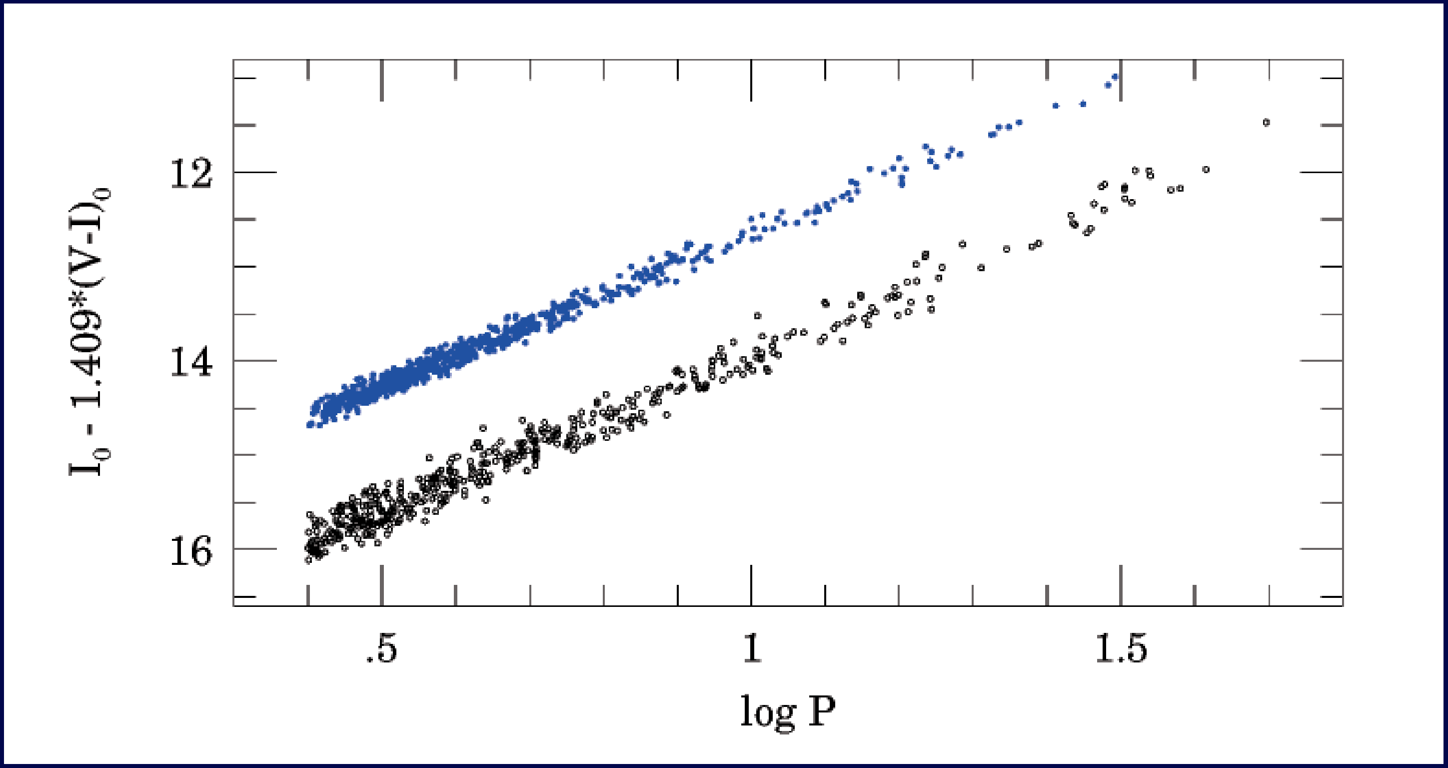

The Cosmic Ladder

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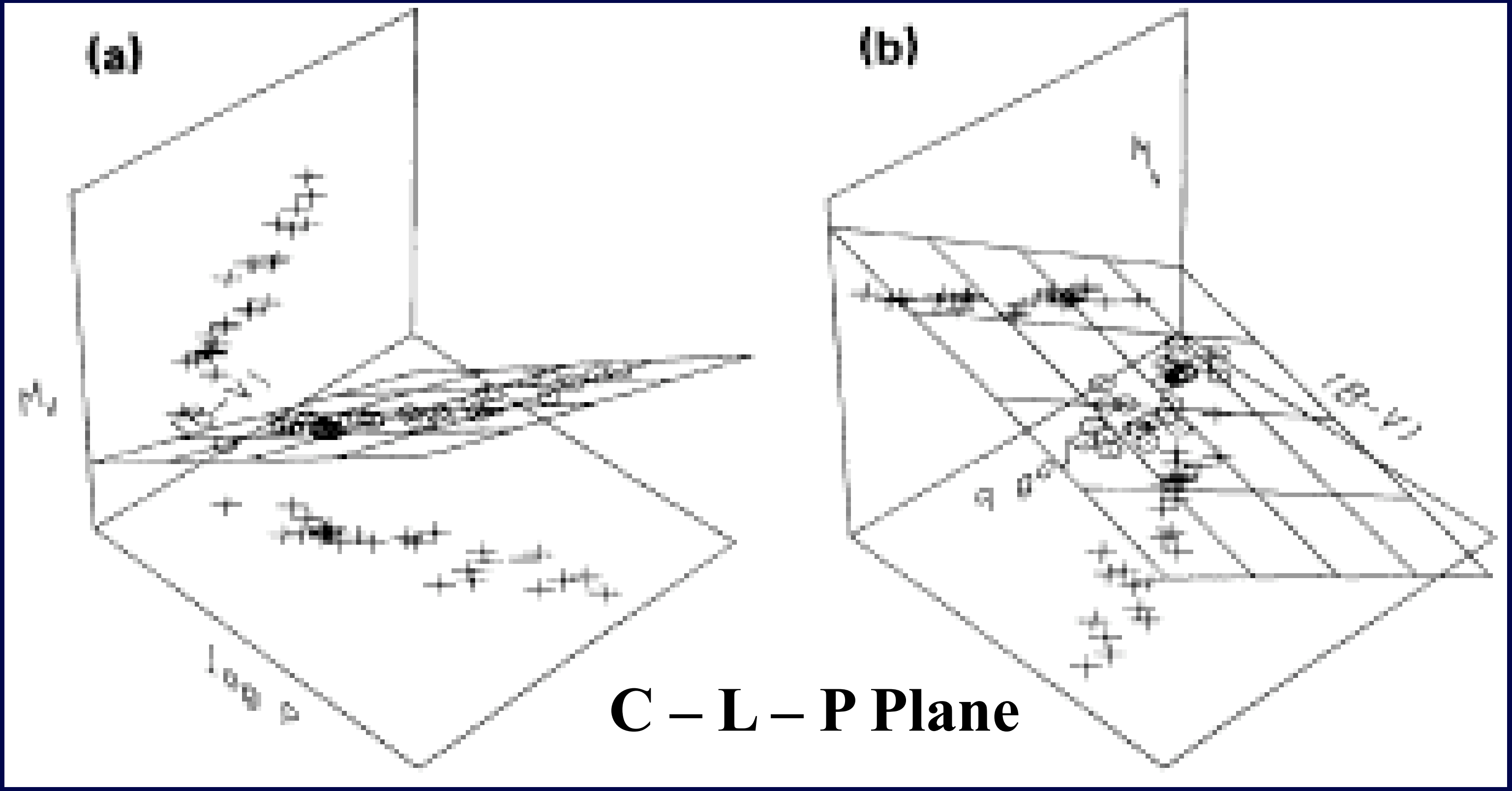


The Cosmic Ladder

- **Find correlated observables:**

  Period – Luminosity **variable stars (Cepheids, RR-Lyr, …)**

- **Use variable stars to find distances to distant galaxies**

- **Find other correlated observables:**

  - **Tully – Fisher** **Spiral galaxies** $L \propto v_r^4$

Tully-Fisher

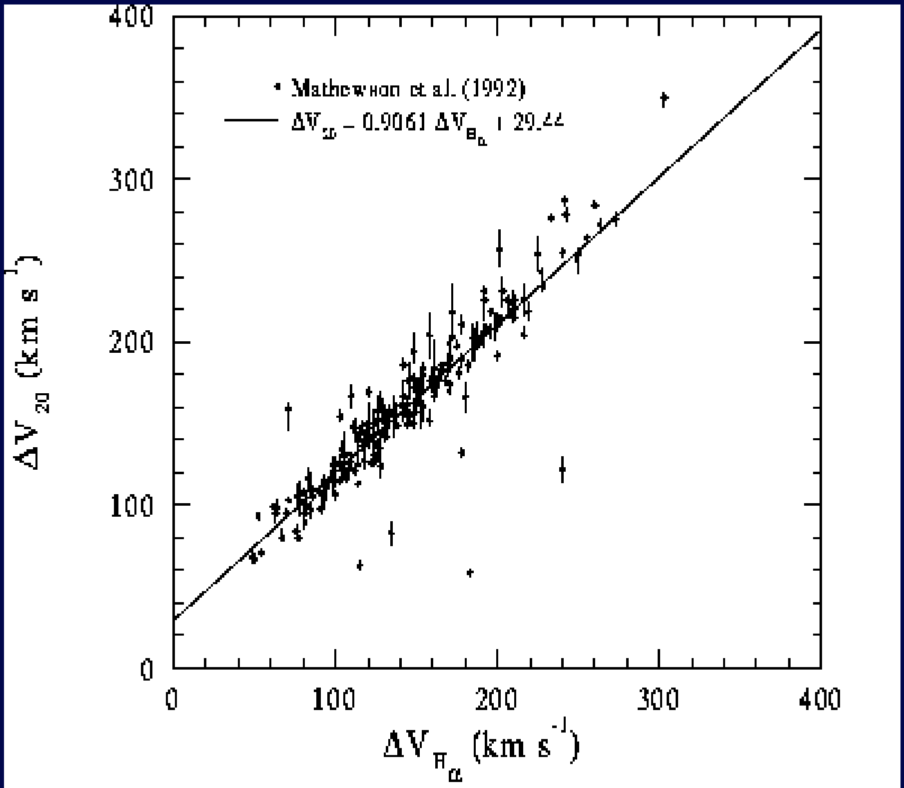
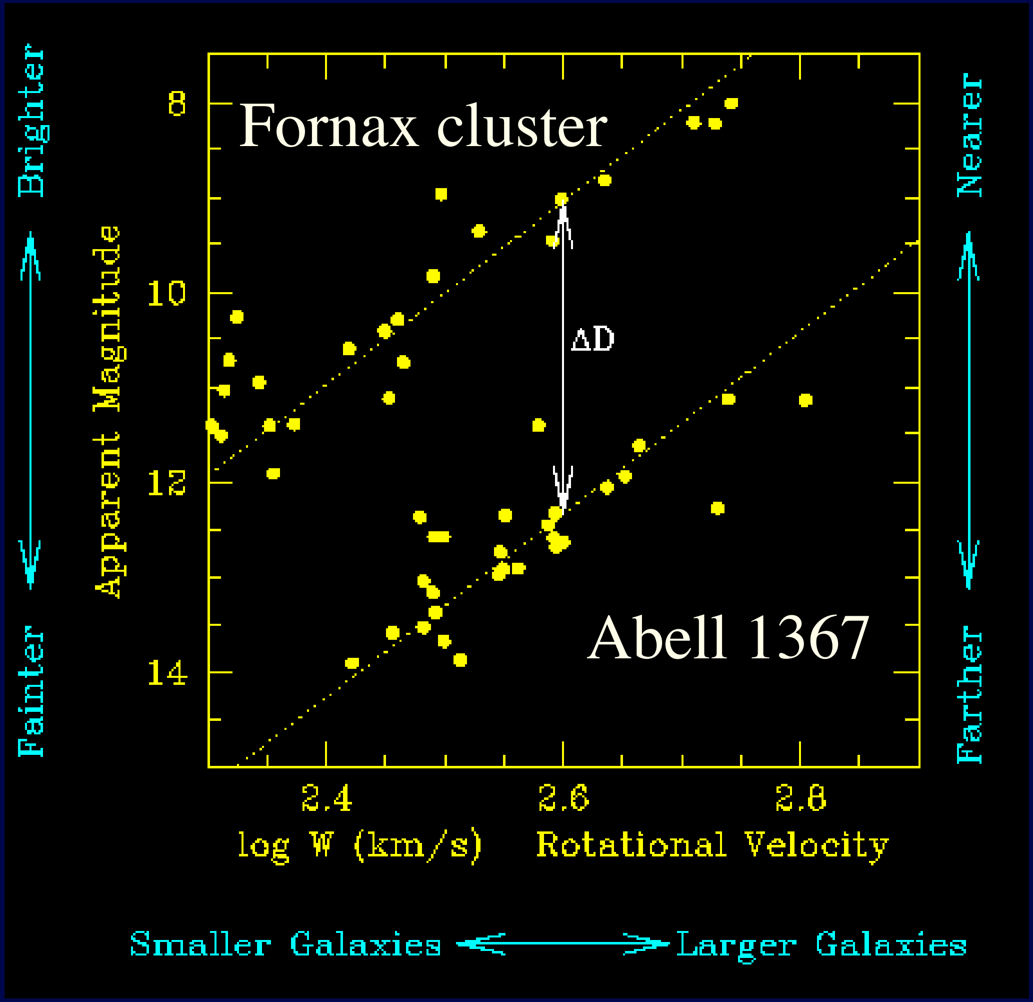


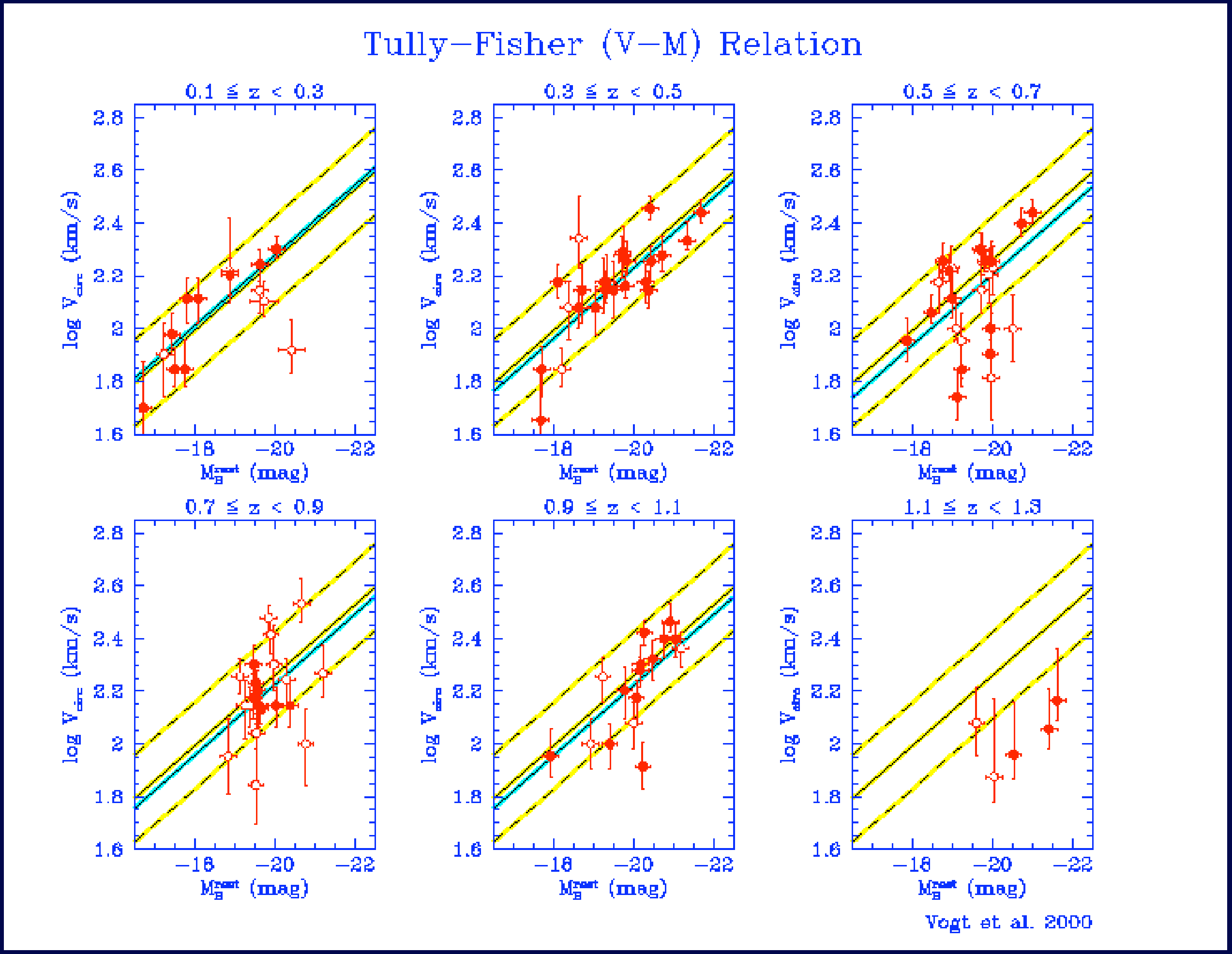
Figure 8.1: HI half-linewidth versus H_α rotation velocity for a sample of 204 nearby galaxies. Data taken from Mathewson *et al.* (1992)

Tully-Fisher



ΔD = *relative* difference between the distances of the two clusters

Tully-Fisher



Tully-Fisher

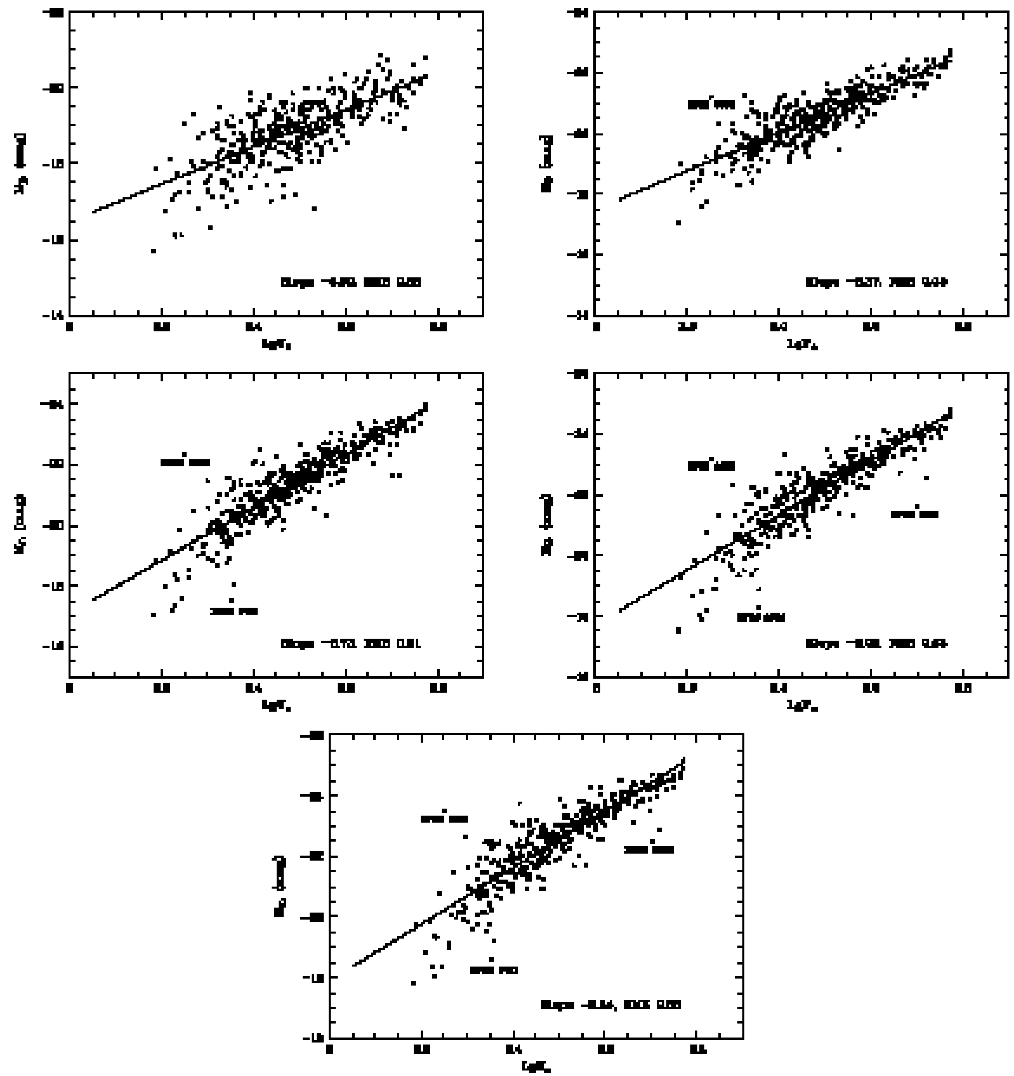


Fig. 8. Tully-Fisher relation for the RFGC galaxies in different photometric bands.

The Cosmic Ladder

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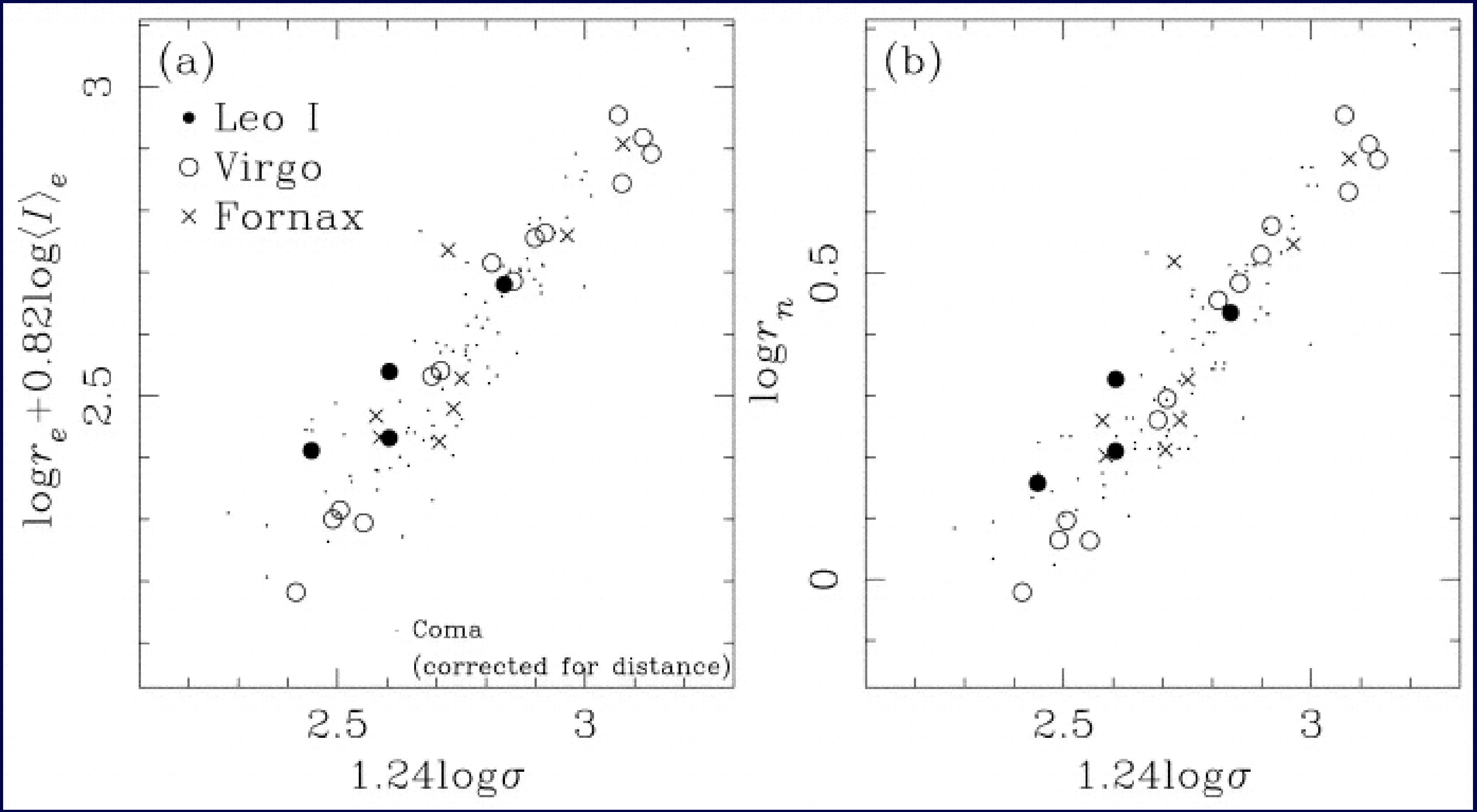
  - $D_n - \sigma$ Elliptical galaxies   L@$B_i \propto \sigma_v^4$

$$D_n \propto r_c < I >_c^{0.8}$$

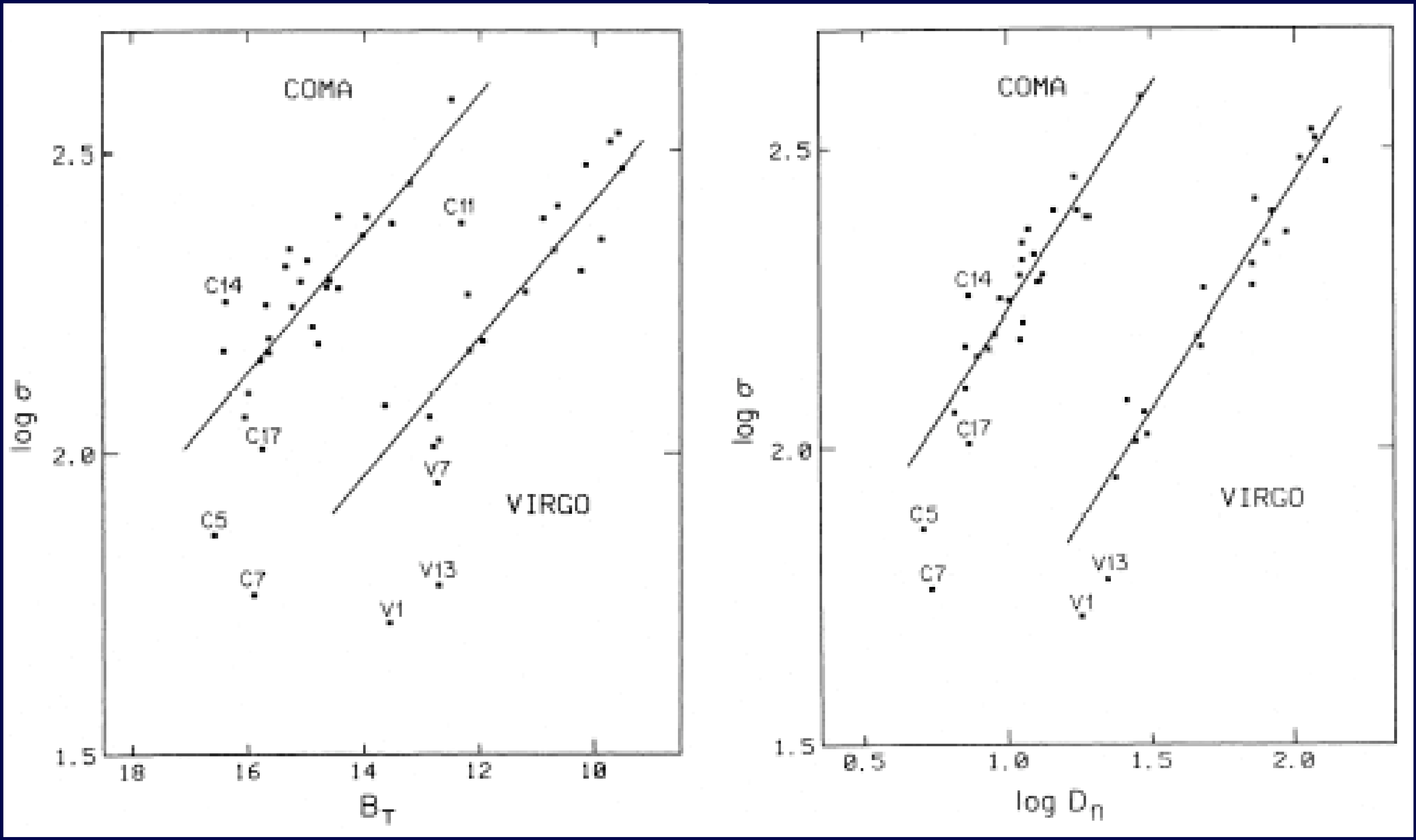
$$\log D_n = 1.333 \log \sigma + \text{constant}$$

$D_n - \sigma$



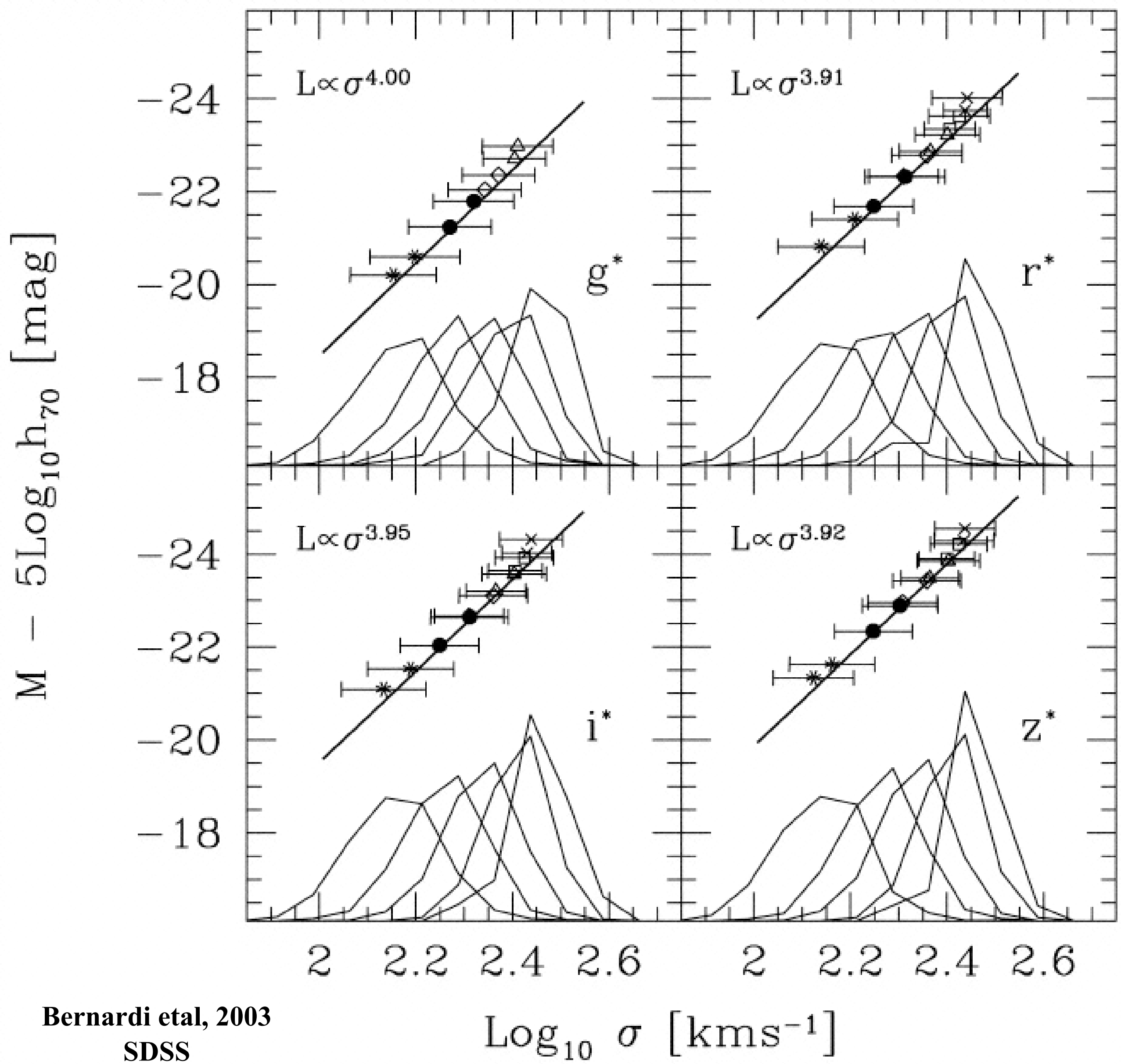
(a)
Leo I
Virgo
Fornax
Coma
(corrected for distance)
3
2.5
2.5
3
1.24logσ
logr_e+0.82log⟨I⟩_e
(b)
logr_n
0.5
0
2.5
3
1.24logσ

$D_n - \sigma$



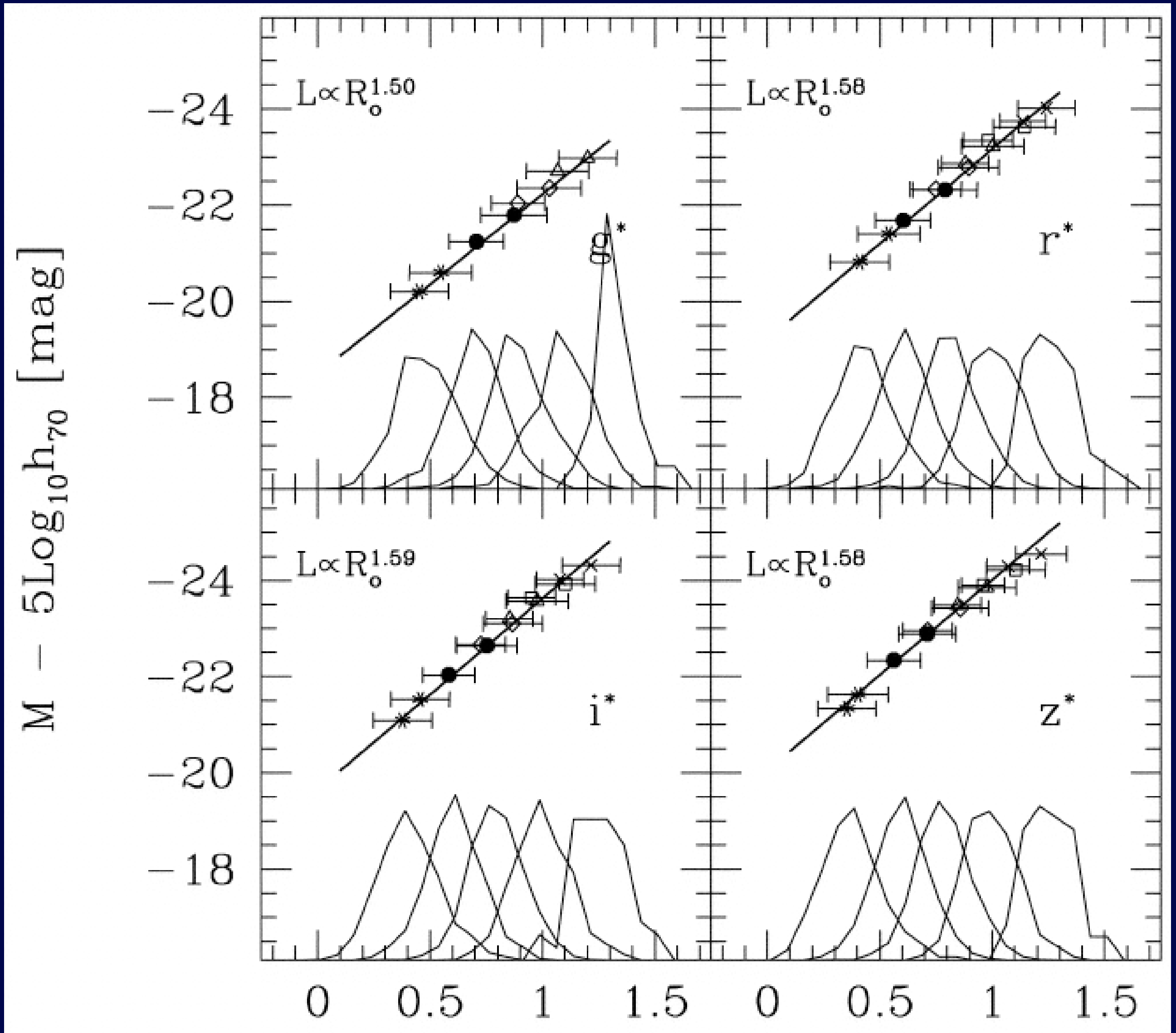
COMA
VIRGO
log σ
2.5
2.0
1.5
18
16
14
12
10
B_T
C14
C17
C11
C5
C7
V7
V13
V1
0.5
1.0
1.5
2.0
log D_n

$D_n - \sigma$



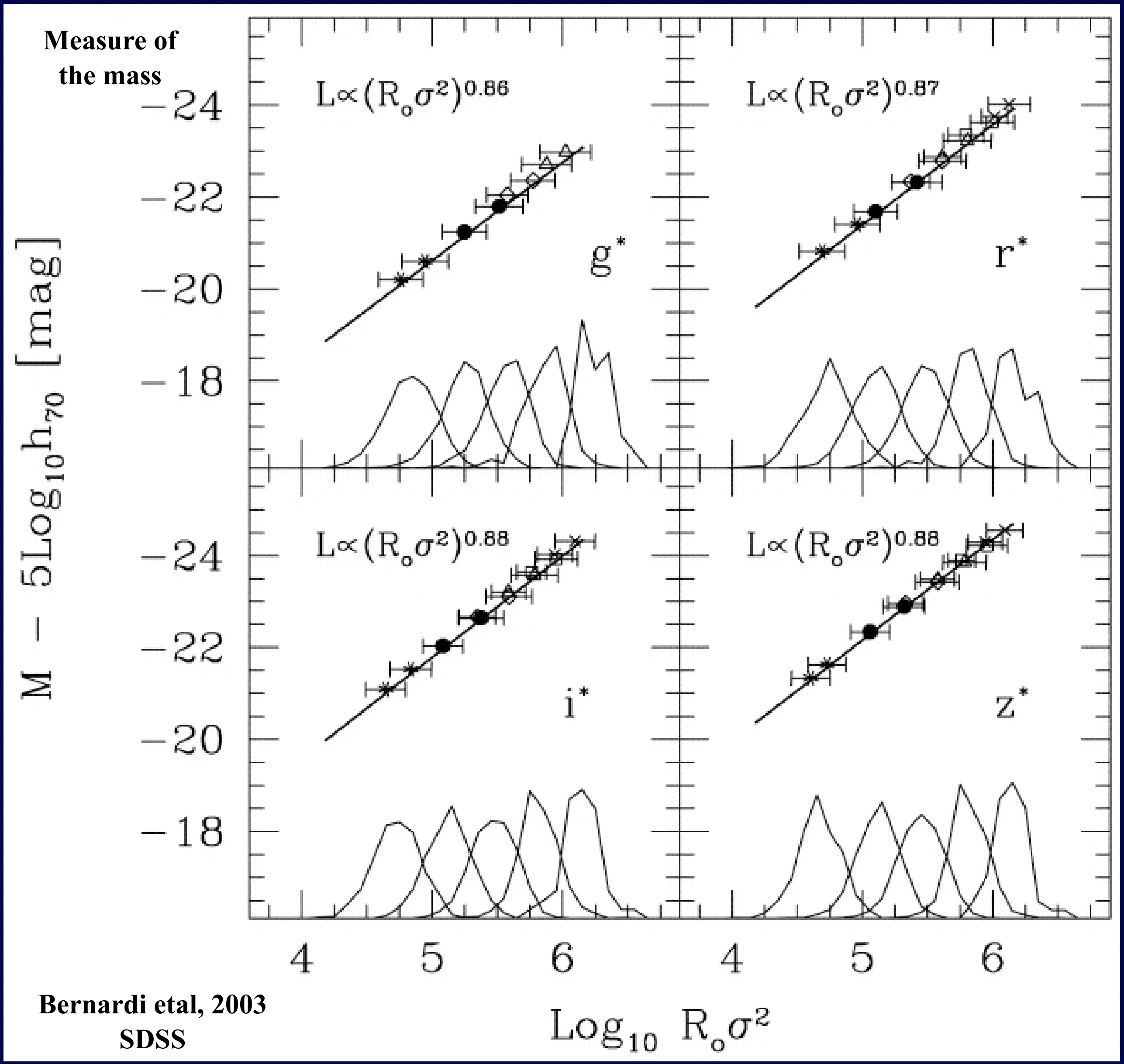
Bernardi etal, 2003
SDSS

$D_n - \sigma$



Bernardi etal, 2003
SDSS

$D_n - \sigma$


Measure of
the mass
L∝(R$_o\sigma^2$)$^{0.86}$
L∝(R$_o\sigma^2$)$^{0.87}$
L∝(R$_o\sigma^2$)$^{0.88}$
L∝(R$_o\sigma^2$)$^{0.88}$
g*
r*
i*
z*
M − 5Log$_{10}$h$_{70}$ [mag]
Log$_{10}$ R$_o\sigma^2$
Bernardi etal, 2003
SDSS

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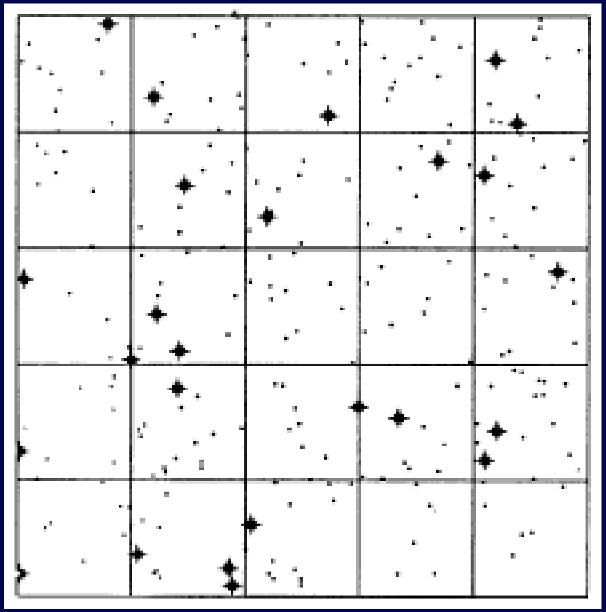
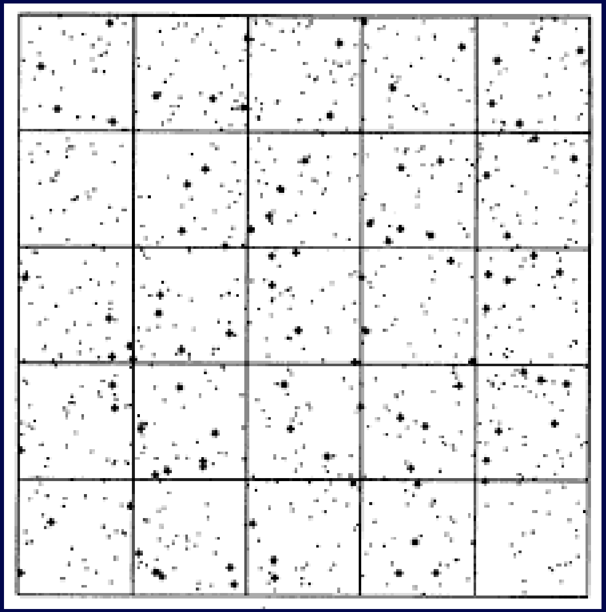
  - $D_n - \sigma$ **Elliptical galaxies** $L@B_i \propto \sigma_v^4$
  - **Surface Brightness Fluctuations**

The Cosmic Ladder

Surface Brightness Fluctuations (SBF)





Measure: flux per pixel f_{av}

rms fluctuations in flux between pixels

Mean: $f_{av} \sim N\, f^*_{av} \sim d^2\, d^{-2} \sim \text{constant}$

Variance: $\sigma^2 \sim N\, (f^*_{av})^2 \sim d^{-2}$

rms: $\sigma \sim d^{-1}$

⇒ A galaxy twice as far is twice as smooth

The Cosmic Ladder

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  - Tully – Fisher Spiral galaxies $L \propto v_r^4$

  - $D_n - \sigma$ Elliptical galaxies $L@B_i \propto \sigma_v^4$

  - Surface Brightness Fluctuations

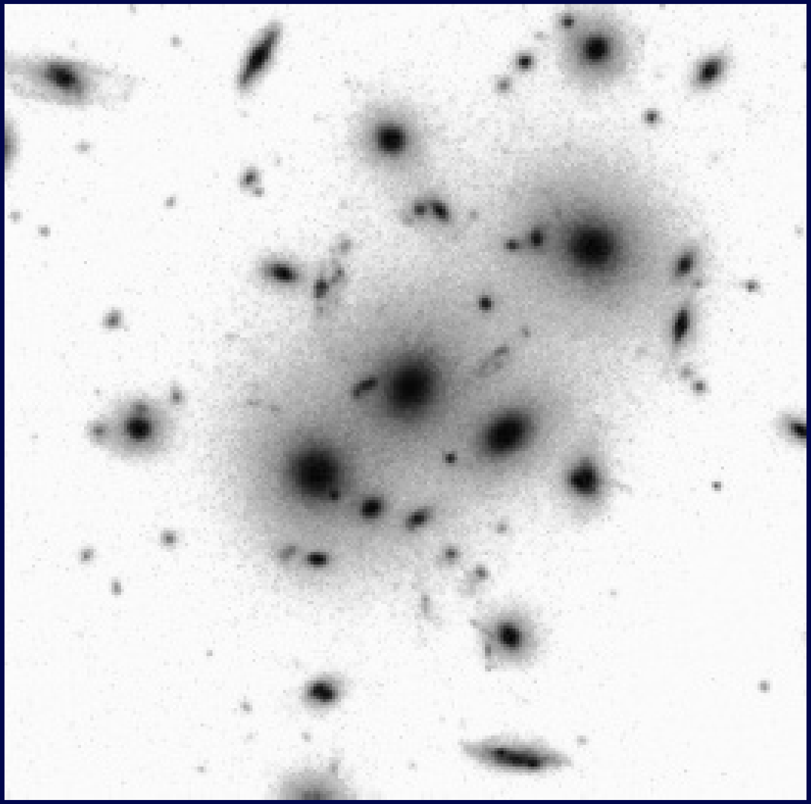
  - Brightest Cluster Galaxy Metric luminosities

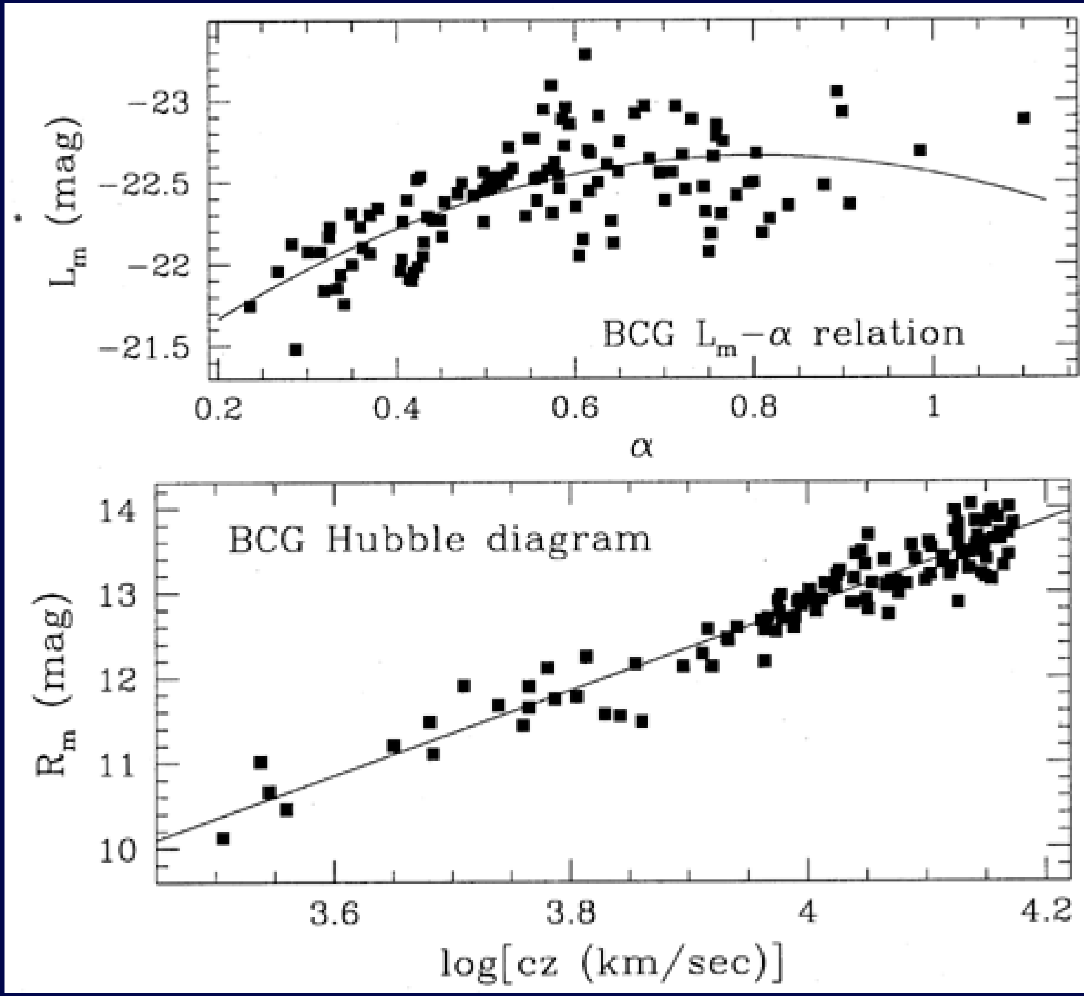
The Cosmic Ladder

- **Brightest Cluster Galaxy**

Metric luminosities

$$\alpha \equiv \frac{dlog(L_m)}{dlog(r)}\bigg|_{r_{10}}$$





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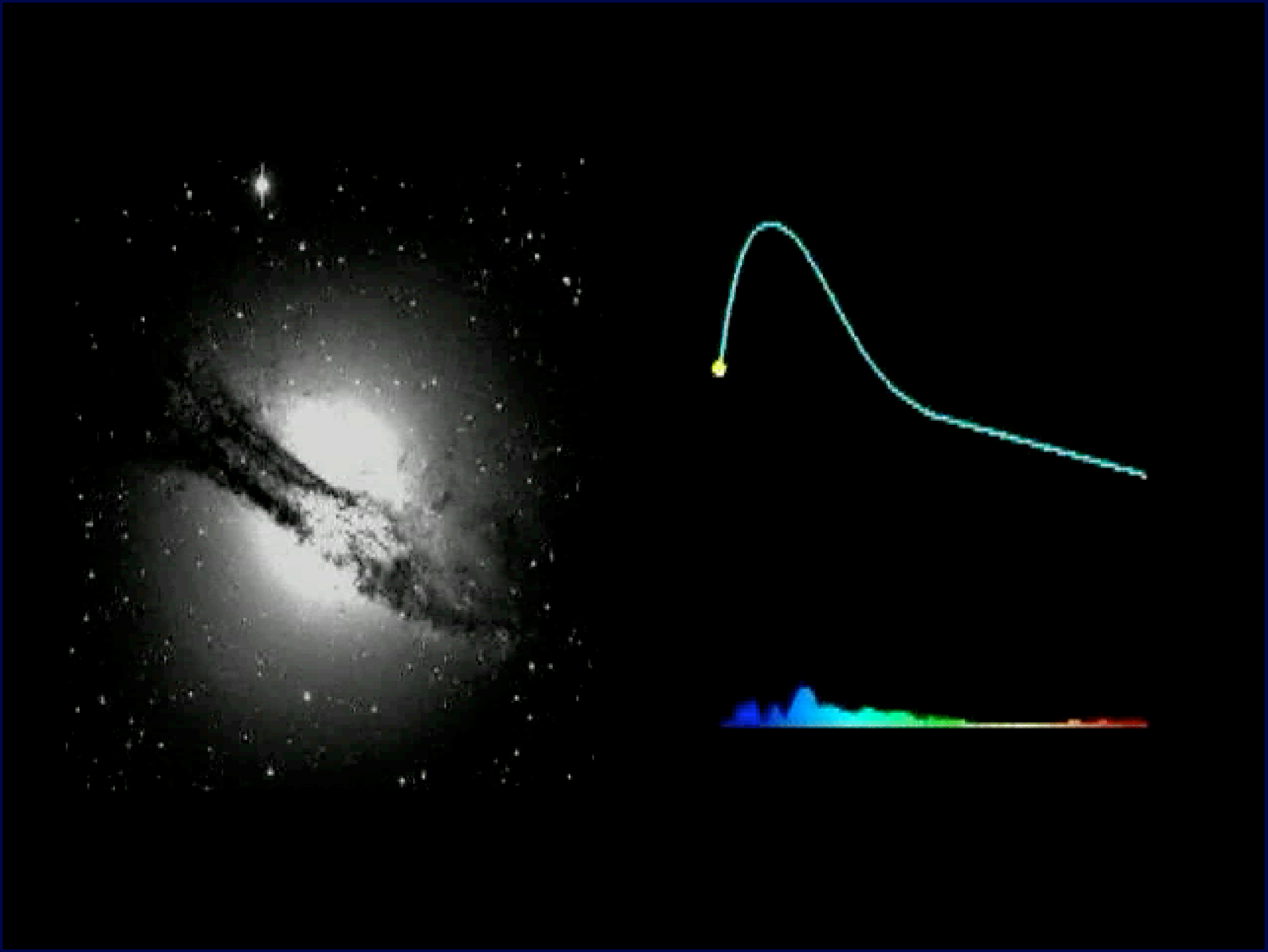
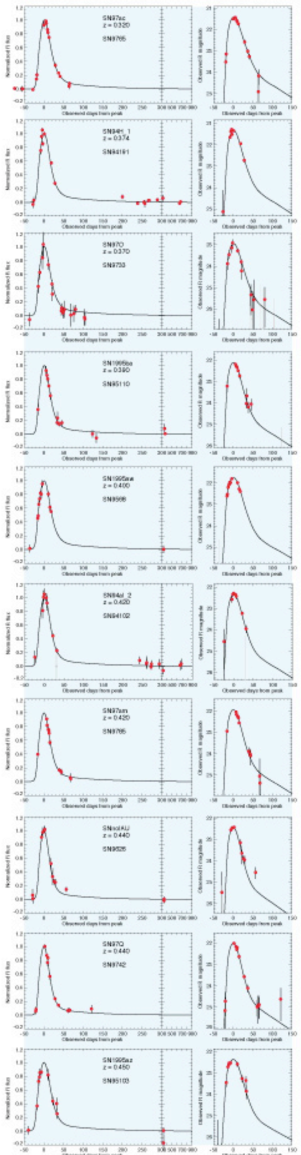
  - **Surface Brightness Fluctuations**
  - **Brightest Cluster Galaxy** Metric luminosities
  - **Supernaovae Type Ia** Light Curve Shapes



Type Ia Supernovae

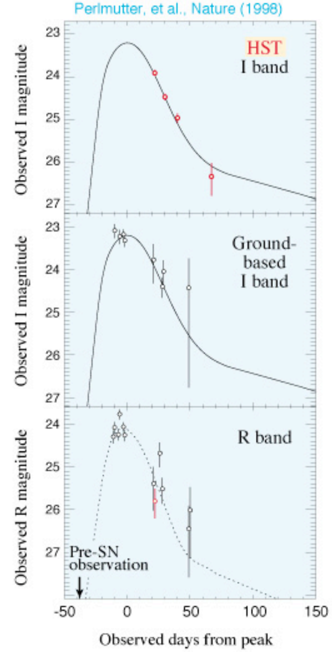
from z = 0.32 ...
observed from the ground



Light Curves

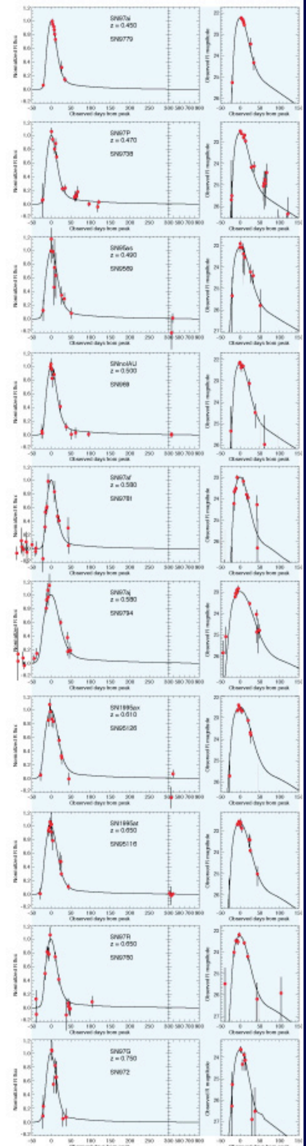
SN 1997ap at z = 0.83
observed from the ground and with the HST

Perlmutter, et al., Nature (1998)



We observe most of the supernovae for approximately two months in both the R and I bands (corresponding approximately to the restframe B and V bands for the median redshift). At high redshifts, a significant fraction of this host galaxy light is within the seeing disk of the supernova, so final observations about one year later are usually necessary to observe (and subtract) the host galaxy light after the supernova has faded. The plots to the left and the right show just the R band light curves for about half of the 40 supernovae that have been completely observed and analyzed so far. The plots above show the highest redshift spectroscopically confirmed supernova, which was observed with the Hubble Space Telescope.

... to z = 0.75
observed from the ground



http://www-supernova.lbl.gov/

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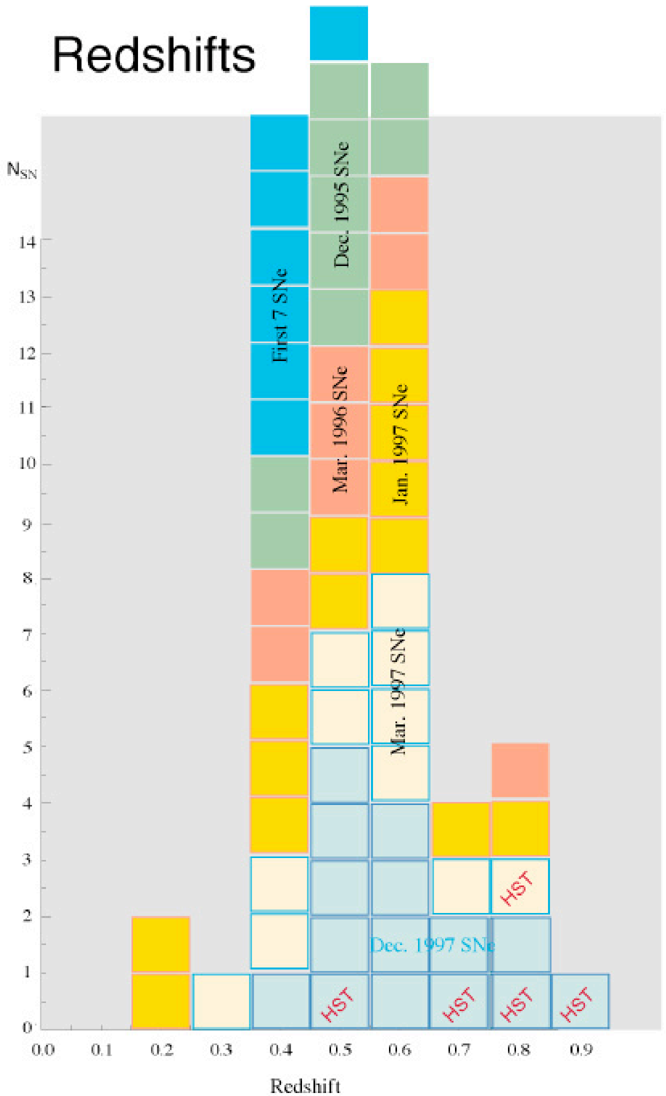

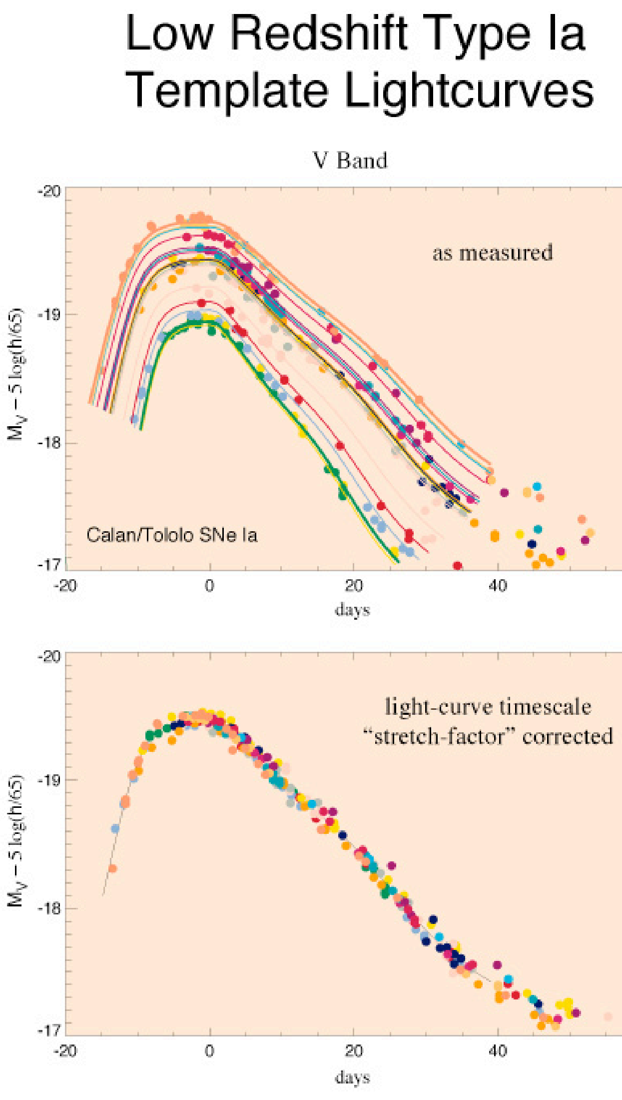

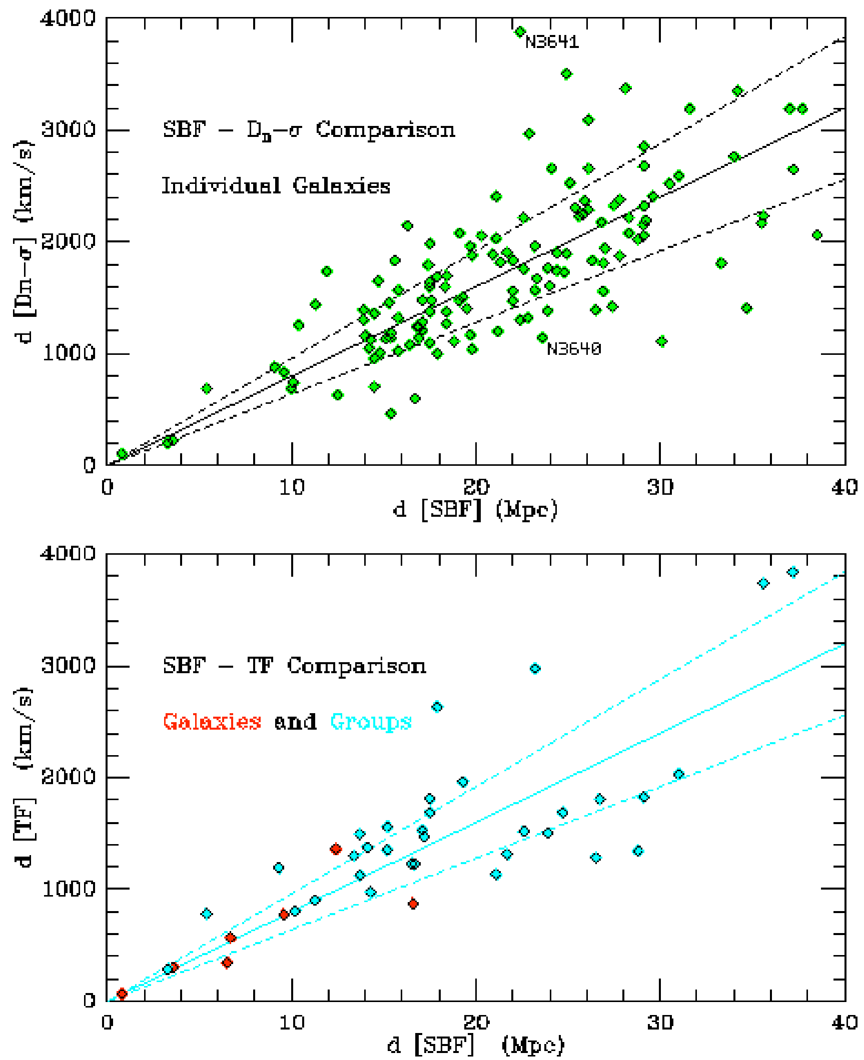

SBF – D_n–σ Comparison
Individual Galaxies
N3641
N3640
d [Dn–σ] (km/s)
d [SBF] (Mpc)
SBF – TF Comparison
Galaxies and Groups
d [TF] (km/s)
d [SBF] (Mpc)

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- **Find correlated observables:**

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- **Use variable stars to find distances to distant galaxies**

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  - **$D_n - \sigma$** Elliptical galaxies $L@B_i \propto \sigma_v^4$
  - **Surface Brightness Fluctuations** $\alpha \equiv d\log(L_m)/d\log(r)\,|r_{10}$
  - **Brightest Cluster Galaxy (BCG)** Metric luminosities
  - **Supernaovae Type Ia (SNIa)** Light Curve Shapes
  - **Sunayev–Zeldovich Effect (SZE)** Cluster distances

SZ Effect

CMB photons Compton scatter on hot electrons in clusters.

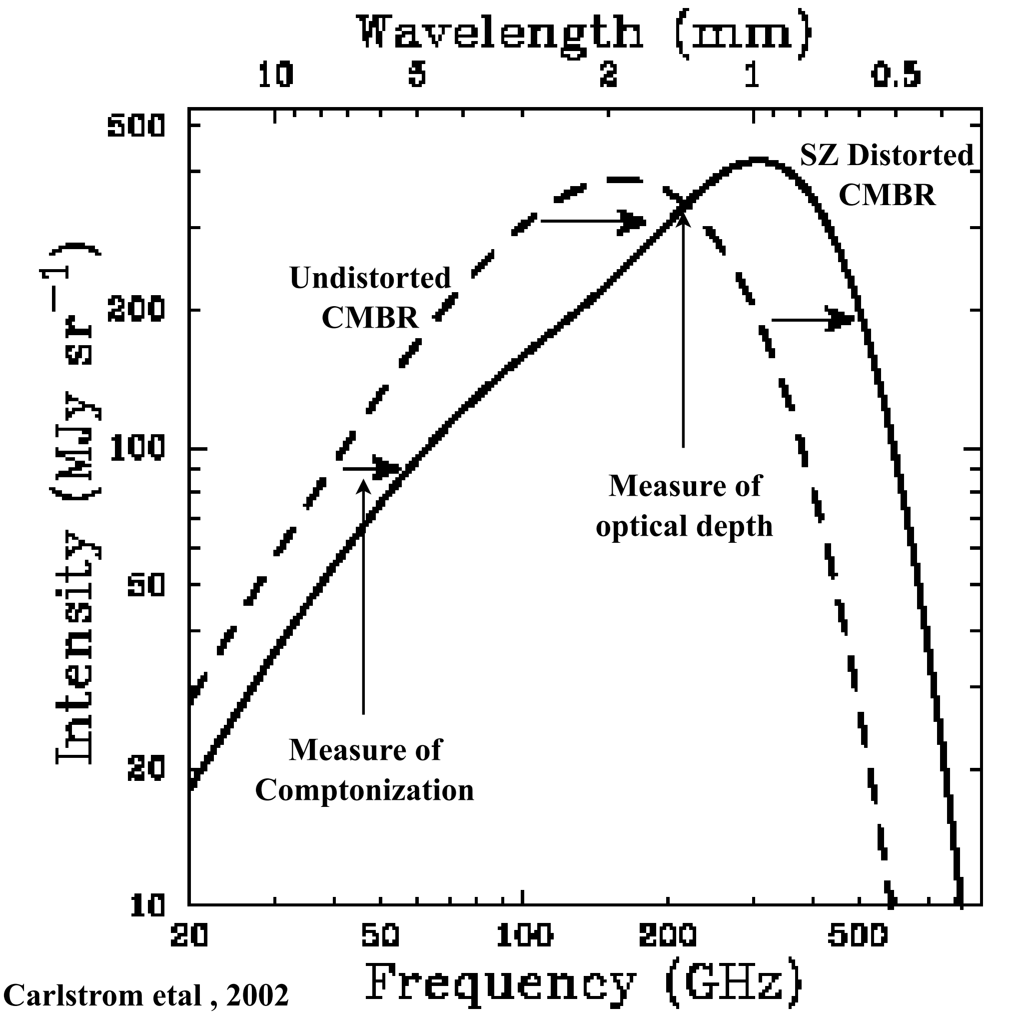
Thermal SZE:

The high T (keV) e^- increase E_γ => non-thermal spectrum

Kinetic SZE:

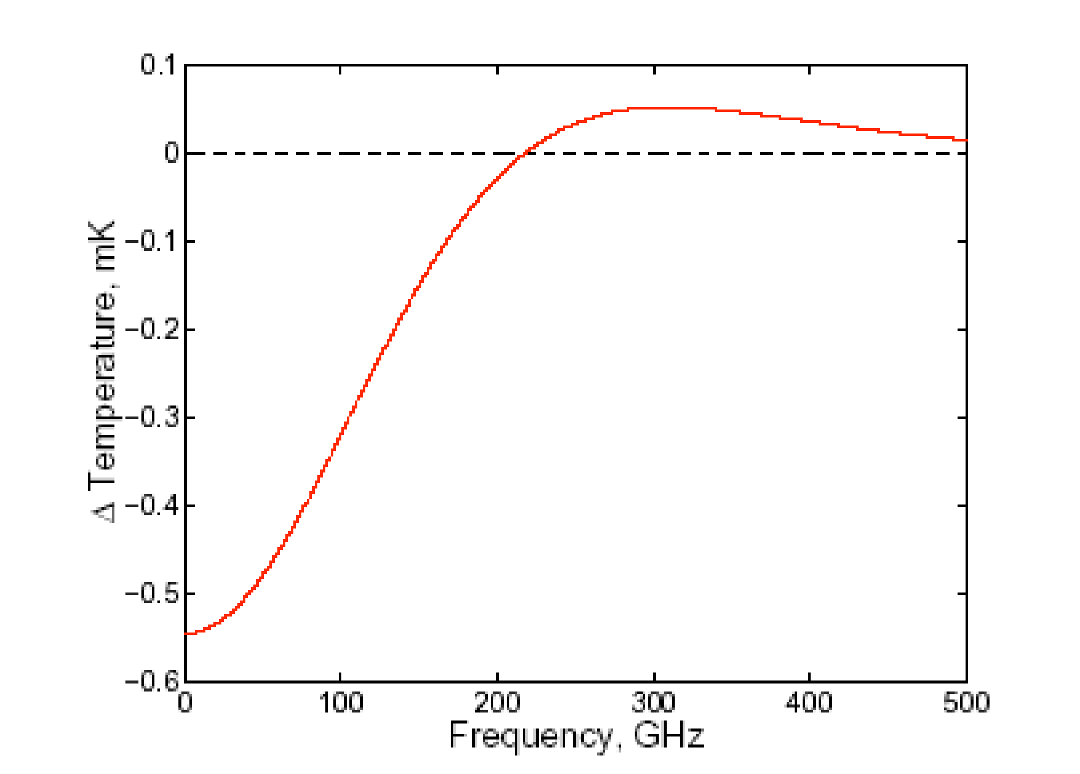
The bulk motion of the cluster red- or blue-shifts scattered γ

SZ Effect



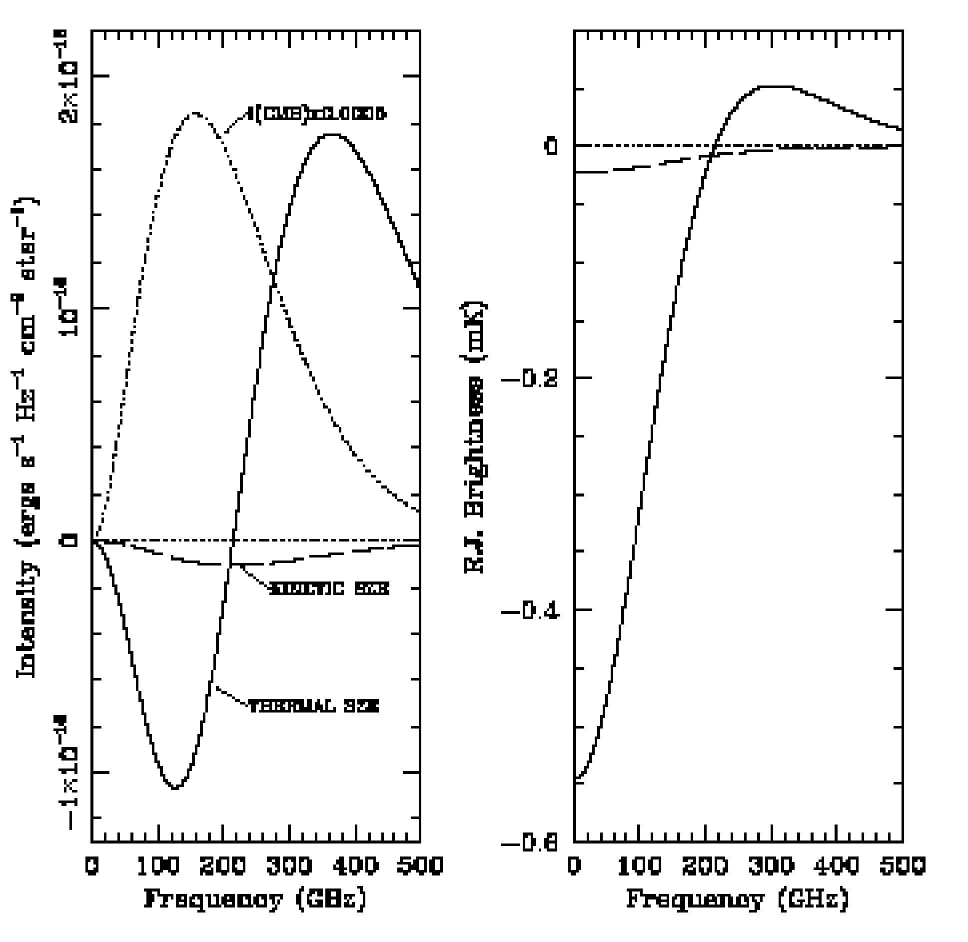
Carlstrom etal , 2002

SZ Effect



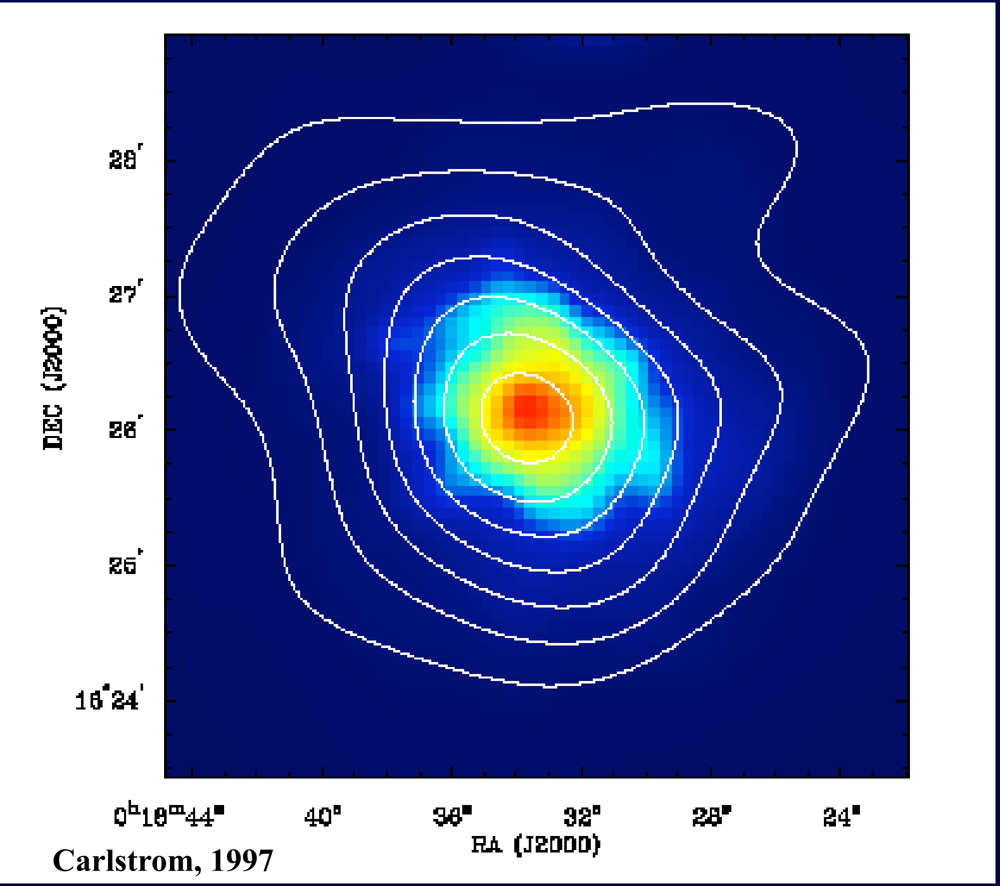
SZE Spectral Distortion of the CMBR due to hot ionized gas associated with a cluster of galaxies

SZ Effect



$T_{e^-} = 10kev \qquad y = 10^{-4} \qquad v_{pec} = 500km/s$

SZ Effect



SZE (contours) and X-ray emission (colors) due to hot gas in cluster 0016+16 (z=0.5455) with $L_{cl} \approx 10^{12} L_{\odot}$.

The cluster appears as a hole in the CMBR

To study the velocity field we first look at

Bulk Flows

At great distances we cannot measure the distances accurately

e.g. At 10,000 km / s (100 h^{-1} Mpc)

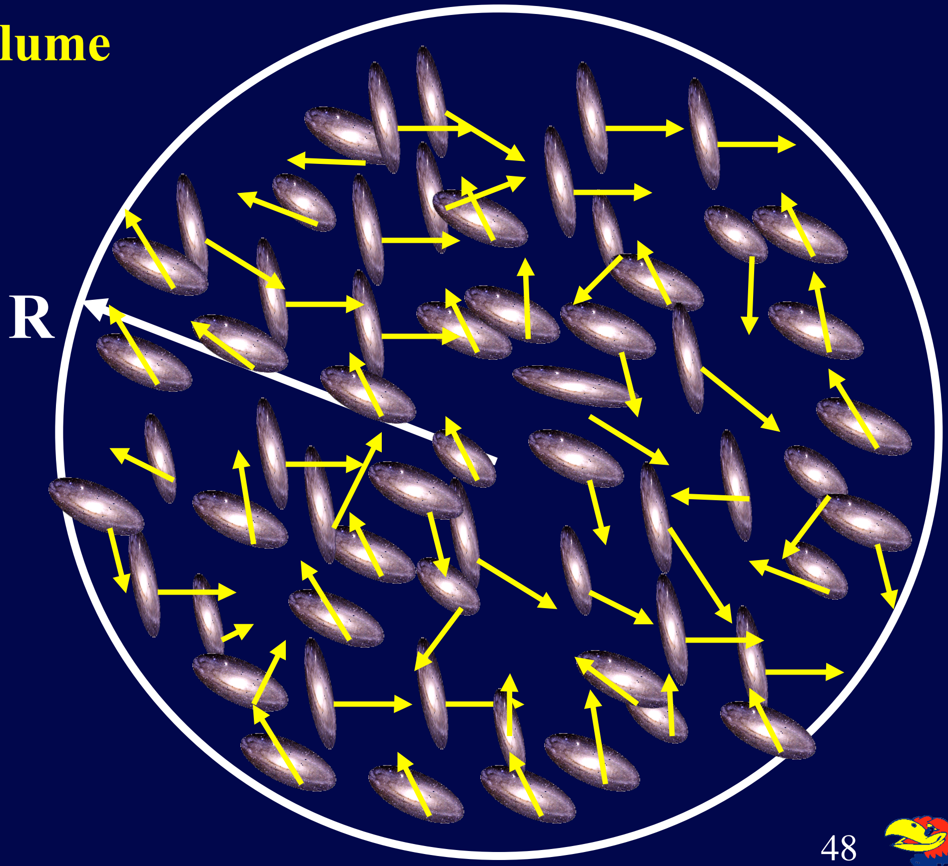
uncertainty of 10% → 1,000 km / s

We want to measure peculiar velocities of

≤ 500 km / s

Combine data to find net motion of a volume

Beat down the error by √N



As R becomes large, expect $v_p \rightarrow 0$

Test homogeneity

Applications

1975 – Rubin & Ford: **Sc Galaxies ($H_o r \leq 10{,}000$ km/s)**

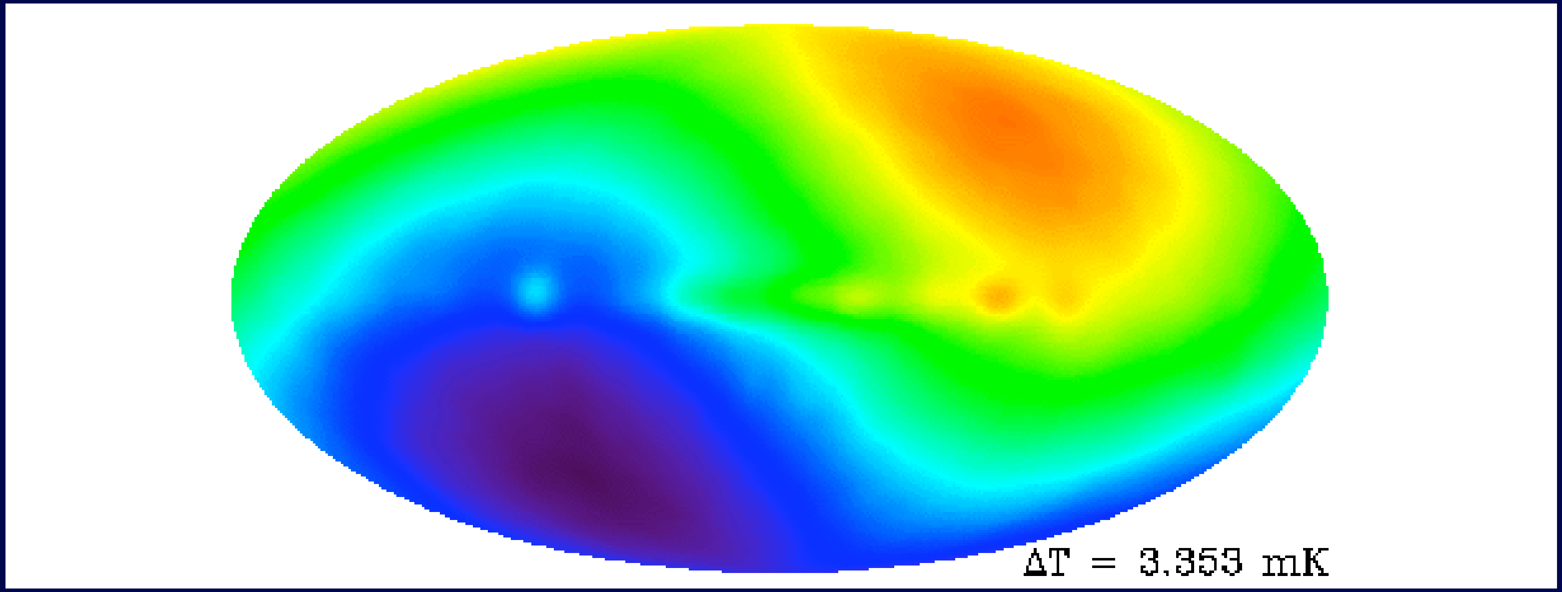
$V_{LG} \sim 550$ km/s

Applications

1975 – Rubin & Ford: Sc Galaxies ($H_o r \leq 10{,}000$ km/s)

$V_{LG} \sim 550$ km/s

1976 – CMB Dipole: $V_{LG} \sim 620$ km/s



Applications

1975 – Rubin & Ford: Sc Galaxies ($H_o r \leq 10{,}000$ km/s)

$V_{LG} \sim 550$ km/s

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1987 – 7 Samurai: $D_n - \sigma$ ($H_o r \leq 6{,}000$ km/s)

$V_{7SIF} \sim 550$ km/s (Great attractor!)

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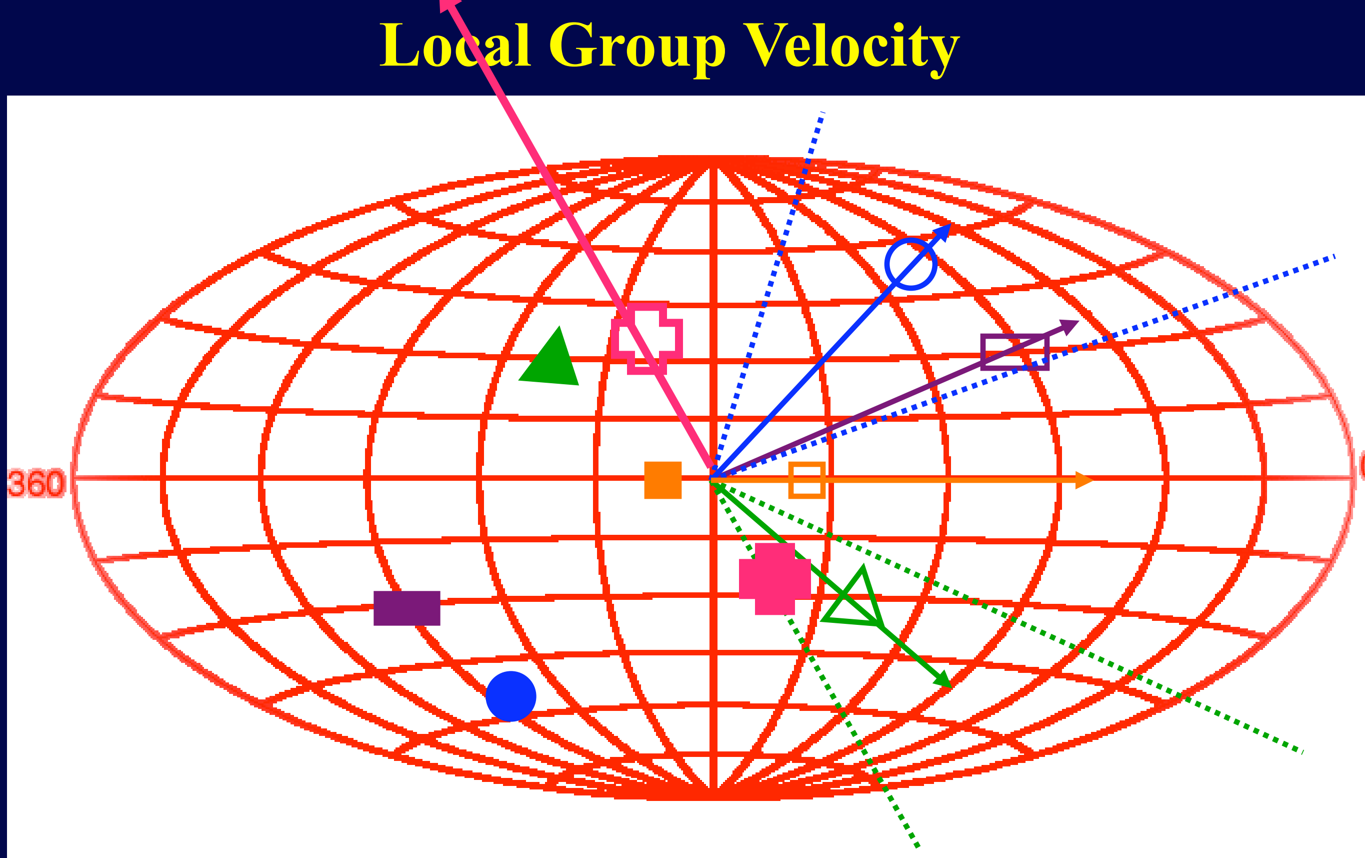
1993 – RPK SN Ia ($H_o r \leq 10{,}000$ km/s)

$V_{SNIF} \sim 400$ km/s (No attractor?)

Local Group Velocity



Survey	l	b	v_p
V_{CMB}	271^o	$+29^o$	620 km / s
V_{LP}	220^o	-28^o	561 ± 284 km / s
V_{RPK}	260^o	$+54^o$	600 ± 350 km / s
V_{SMAC}	195^o	0^o	700 ± 250 km / s
V_{LP10k}	173^o	$+63^o$	1000 ± 500 km / s
V_{SC}	180^o	0^o	100 ± 150 km / s

Recent Large-Scale Bulk Flow Results

Survey	Method	N	Depth km/s	V km/s	Random err (km/s)	l	b
LP	BCG	119	8400	830	220	330	39
SC	TF	63	7000	80	100	290	20
Willick	TF	15	11000	1100	450	270	27
SMAC	FP	56	6000	650	180	260	-4
EFAR	FP	49	9300	650	350	50	10
SNIa	SNIa	65	10000	530	200	313	9

New: Tonry (2003) 6000 km/s < Hd < 30000 km/s

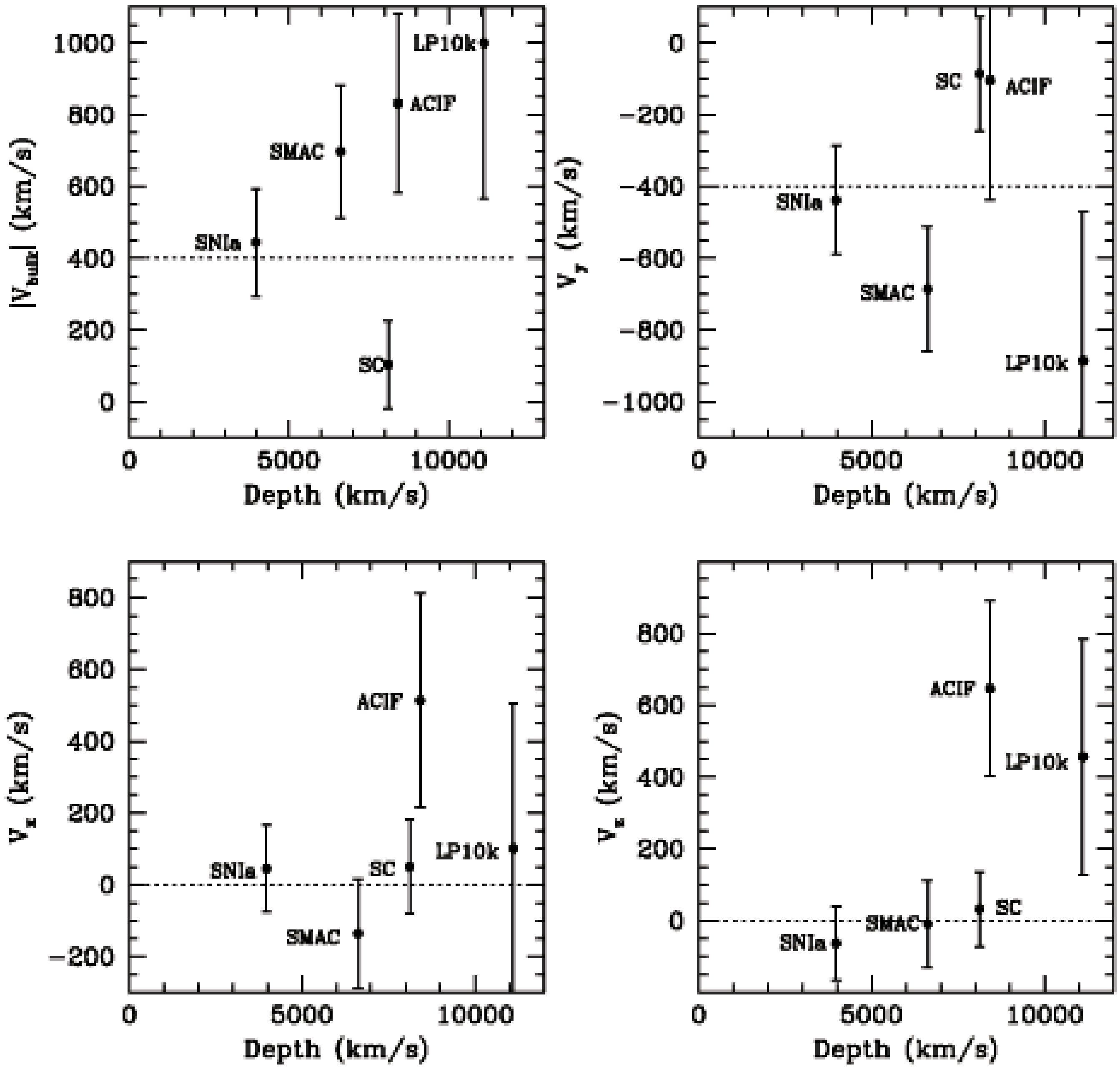
Errors Including Sampling

.. following analysis of Kaiser, Watkins & Feldman

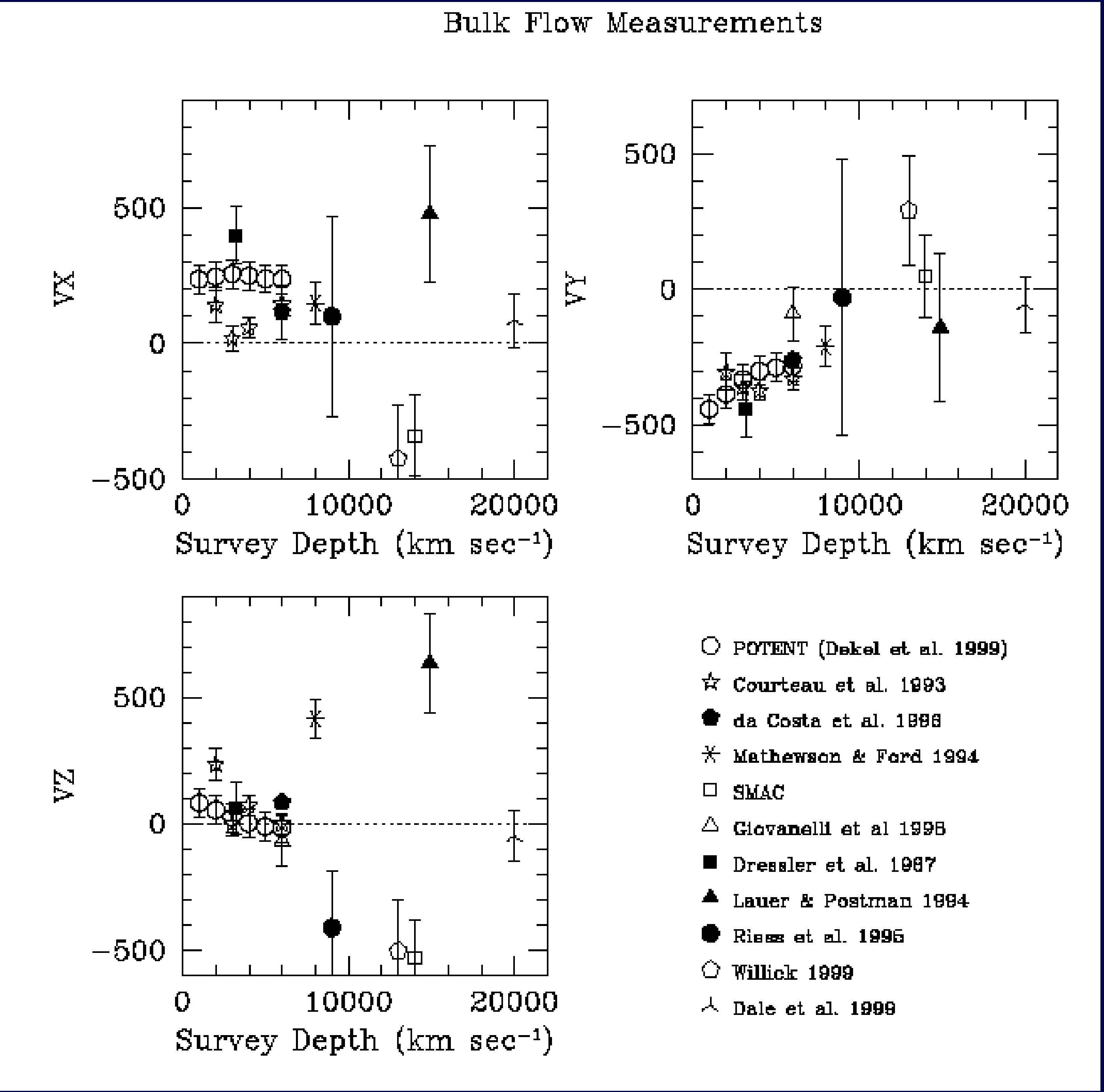
Survey	Method	V km/s	Random err (km/s)	Sampling err (km/s)	l	b
LP	BCG	830	220	110	330	39
SC	TF	80	100	170	290	20
Willick	TF	1100	450	220	270	27
SMAC	FP	650	180	180	260	-4
EFAR	FP	650	350	210	50	10
SNIa	SNIa	530	200	130	313	9

Errors are often as large as or larger than random errors

Hudson, 2003

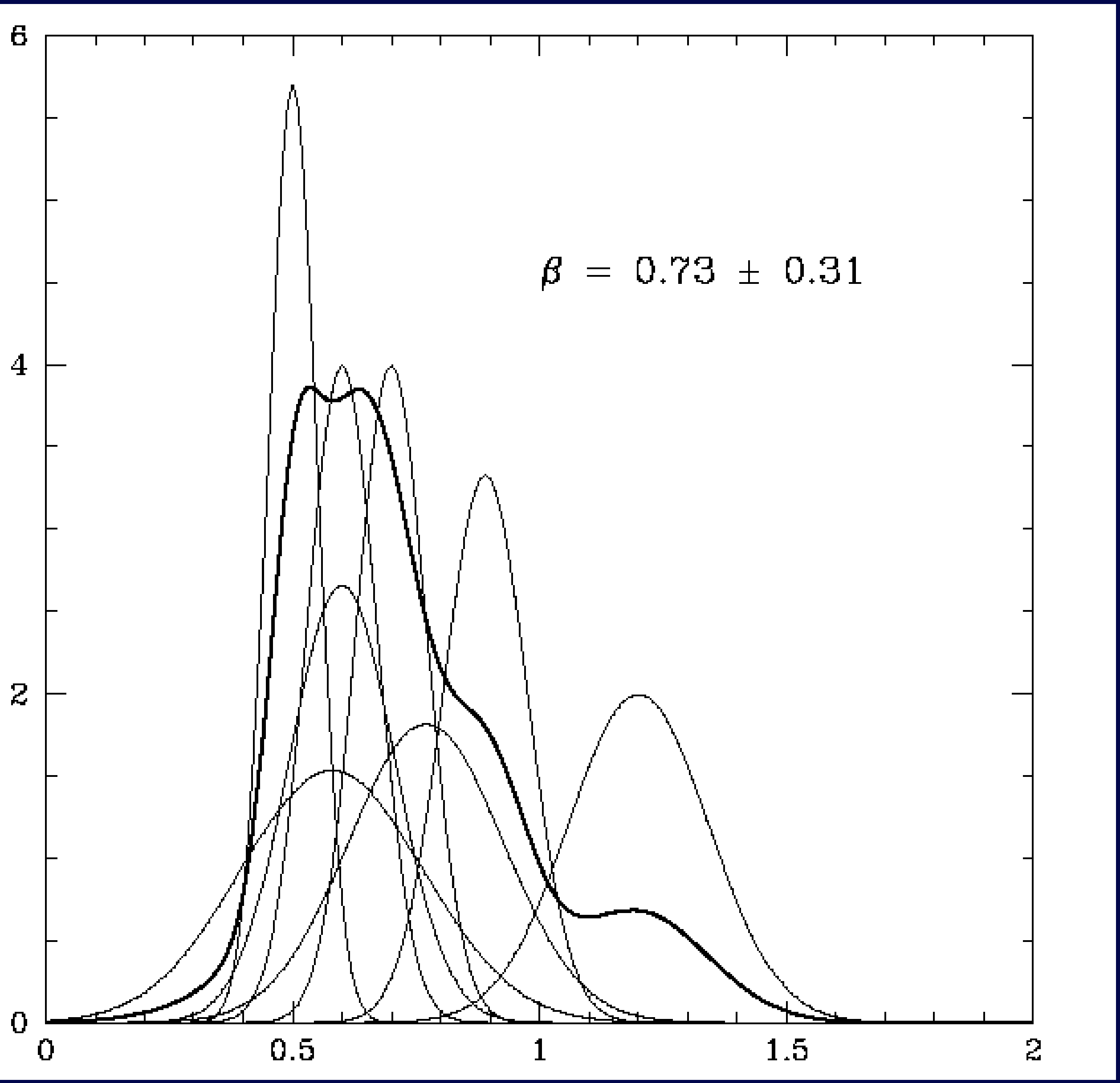


|V_bulk| (km/s)
Depth (km/s)
SNIa
SMAC
ACIF
LP10k
SC
V_y (km/s)
Depth (km/s)
SC
ACIF
SNIa
SMAC
LP10k
V_x (km/s)
Depth (km/s)
ACIF
LP10k
SNIa
SC
SMAC
V_z (km/s)
Depth (km/s)
ACIF
LP10k
SC
SMAC
SNIa



Strauss, 2000

$\beta \equiv \Omega^{0.6} / b$



Systematic effects

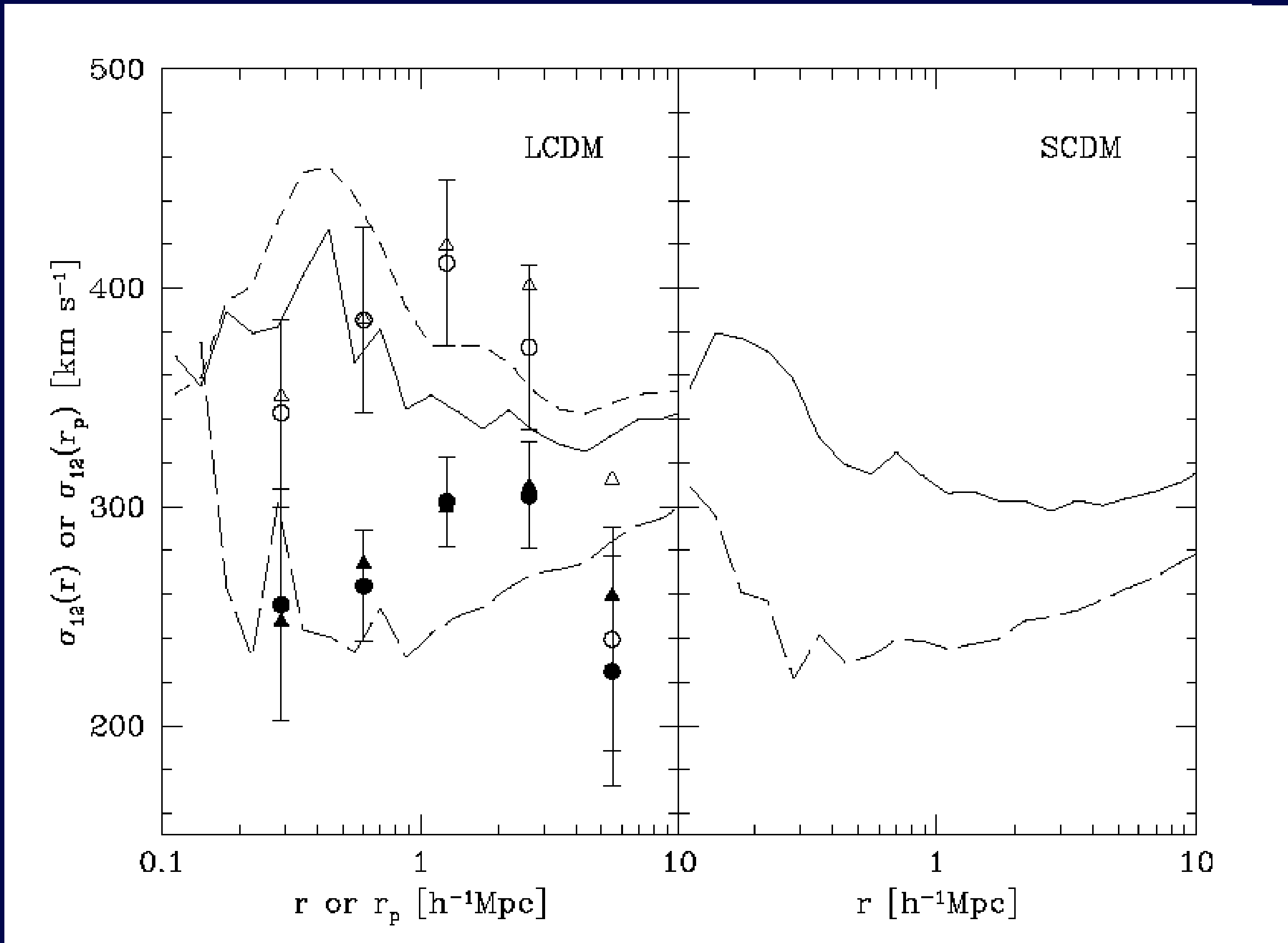
Nonlinear effects

Correlated Errors

Non-trivial biasing

Malmquist biases
standard candles
calibration...

Pairwise velocity Dispersion σ_{12}



Zhao, Jing & Borner, 2002

??? What is going on ???

In large scale observations we look for

Estimators

We try to estimate an underlying quantity

Estimator = True quantity ⊗ Window function

e.g.

$$\tilde{p} = N\int \frac{d^3k}{(2\pi)^3}\, p\left(\vec{k}\right) W\left(\vec{k}\right)$$

$$\tilde{p} = N\int \frac{d^3k}{(2\pi)^3}\, p\left(\vec{k}\right) W\left(\vec{k}\right)$$

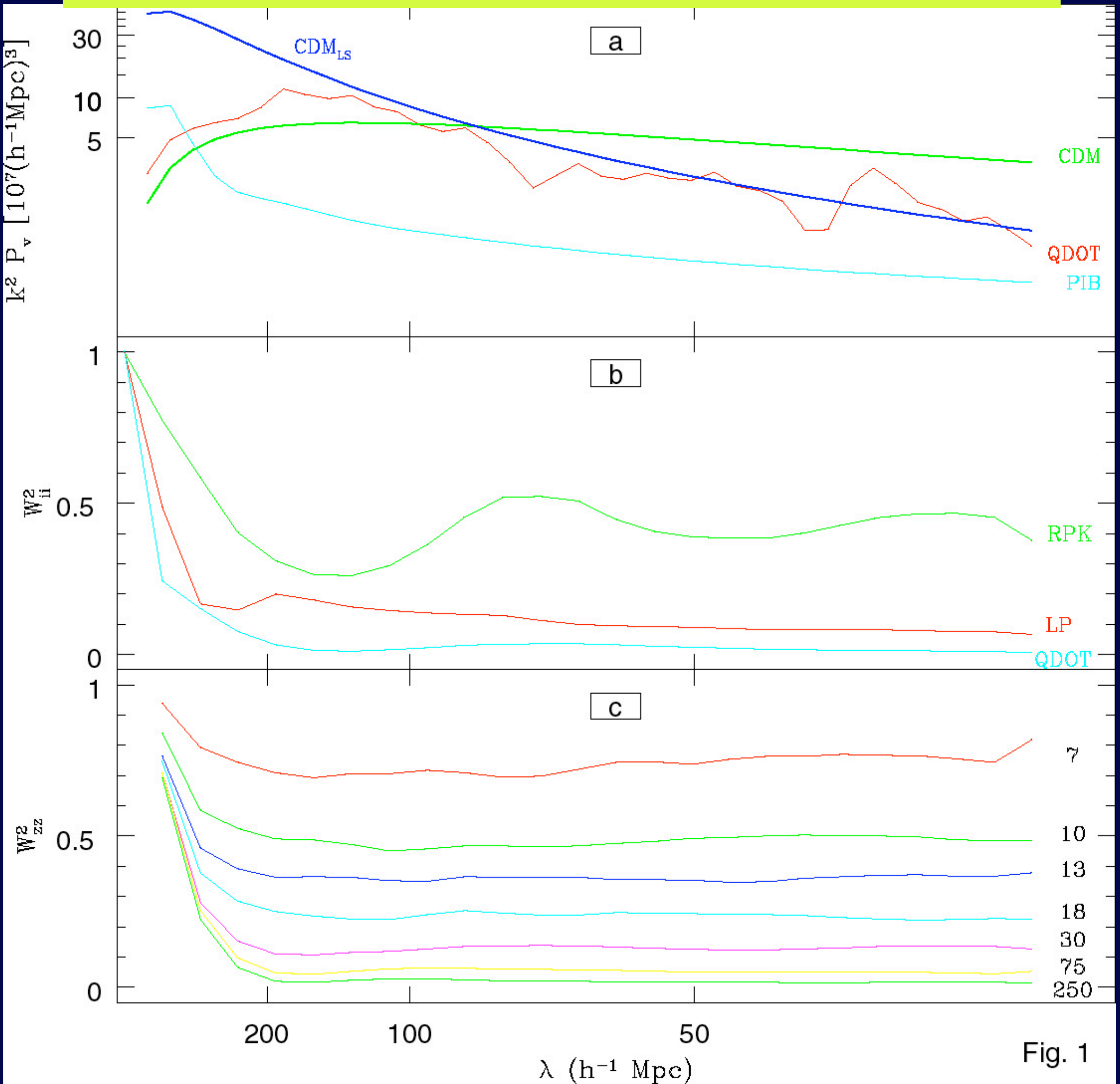


Fig. 1

Coffee Break

Are there any statistics that are consistent across surveys

Mean Pairwise Velocity (v_{12})

"The tendency of galaxies to approach each other"

Close galaxies (e.g. within a cluster, $r_{12} < 3$ Mpc):

Motion about the local center of mass, $v_{12} = 0$

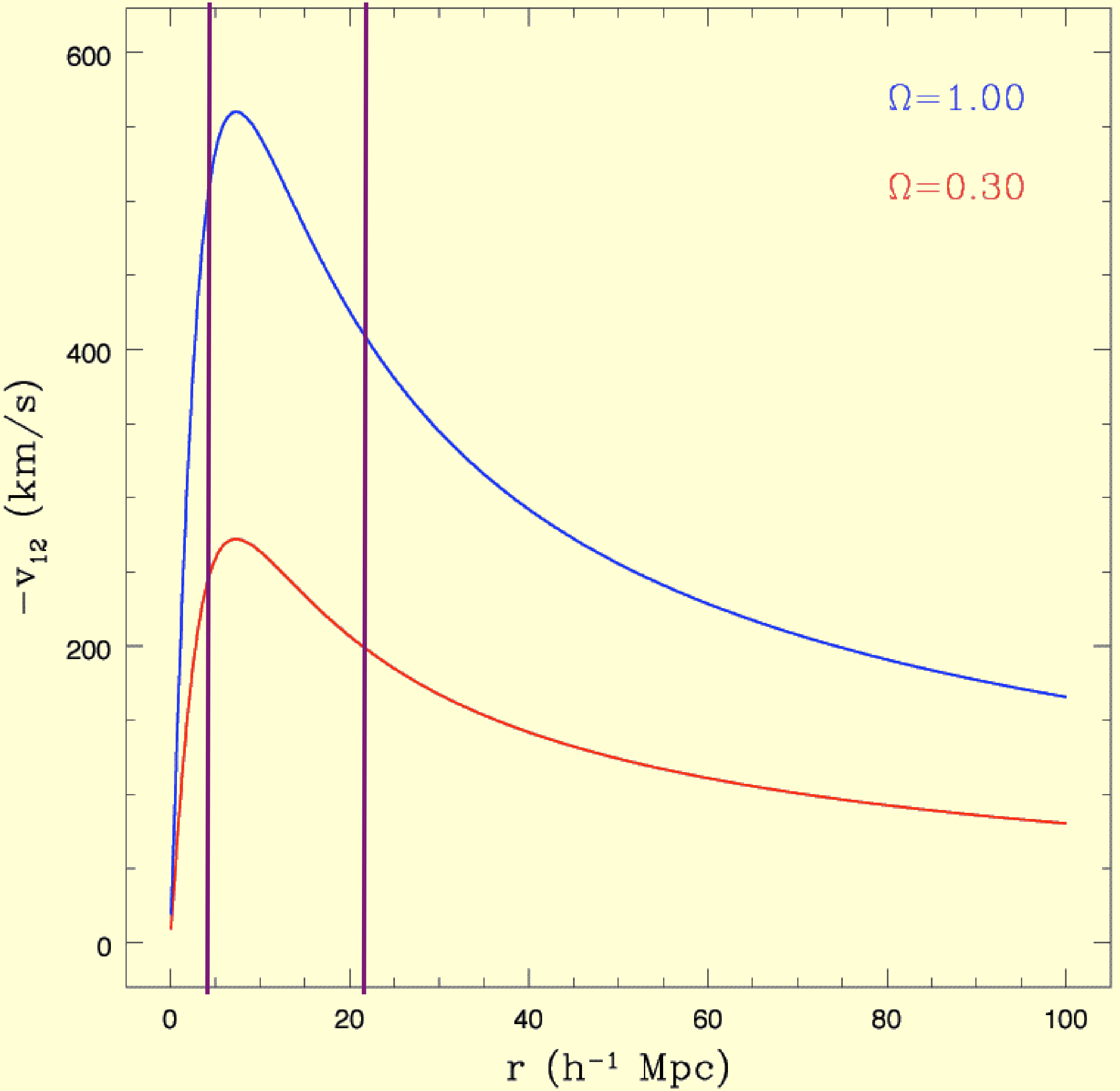
Far Galaxies ($r_{12} > 30$ Mpc)

No attraction ⇒ no correlations, $v_{12} \rightarrow 0$

For galaxies in intermediate scales

$v_{12} < 0$

HAF etal 2003 ApJL. 596 131L
Juszkiewicz etal 2000 Science, 287
Ferreira etal 1999, ApJL 515 L1


600
400
200
0
0
20
40
60
80
100
$-v_{12}$ (km/s)
r (h^{-1} Mpc)
$\Omega=1.00$
$\Omega=0.30$

Can we exploit the mean tendency of well-separated galaxies to approach each other to measure cosmological parameters ?

Consider the dynamical evolution of a collection of particles interacting through gravity.

In the fluid limit the pair-density weighted relative velocity is:

$$\vec{v}_{12}(r) = < \vec{v}_1 - \vec{v}_2 >_\rho = \frac{< (\vec{v}_1 - \vec{v}_2)(1+\delta_1)(1+\delta_2) >}{1+\xi(r)}$$

v_i and $\delta_i = \rho_i/\langle\rho\rangle - 1$ are the peculiar velocities and the density contrast at a point r_i : $r = |\vec{r}_1 - \vec{r}_2|$

and $\xi(r) = < \delta_1 \delta_2 >$

Different than the simple weighted average by $\rho_1\rho_2 / \langle \rho_1\rho_2 \rangle$

The magnitude $v_{12}(r)$ is related to the two-point correlation function $\xi(r)$ through the pair conservation equation in gravitational instability theory.

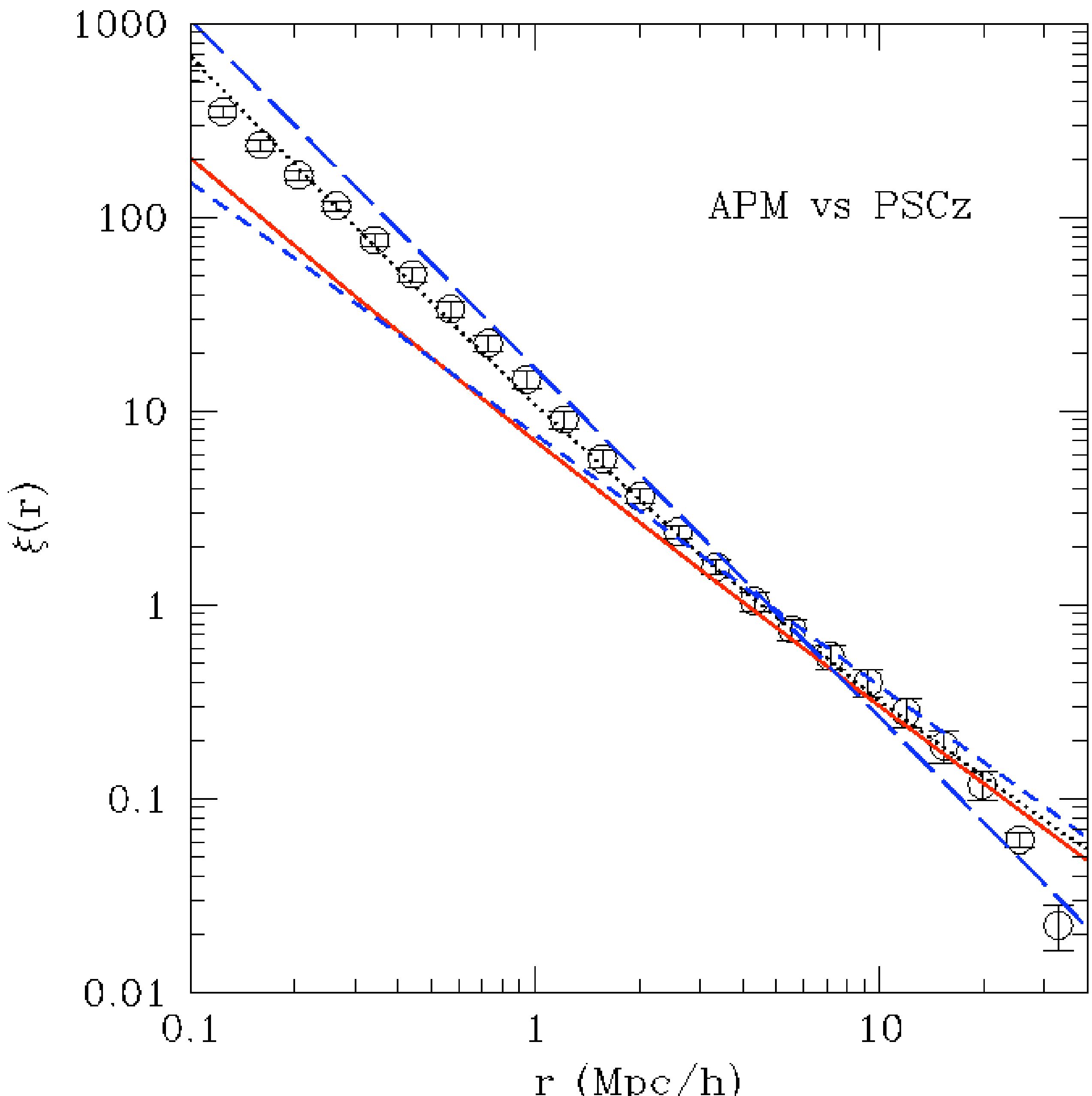
For models with Gaussian initial conditions the solution of the pair conservation equation is well approximated by:

$$v_{12}(r) = -\frac{2}{3} H r \Omega^{0.6} \bar{\bar{\xi}}(r)\left[1 + \alpha \bar{\bar{\xi}}(r)\right],$$

$$\bar{\xi}(r) = \frac{3}{r^3} \int_0^r \xi(x) x^2 dx \equiv \bar{\bar{\xi}}(r)\left[1 + \xi(r)\right]$$

(Juszkiewicz, Springel & Durrer, 1998)

α is a parameter that depends on the logarithmic slope of $\xi(r)$



HAF etal 2003

We cannot estimate v_{12} directly since we only observe the line-of-sight component of the peculiar velocity:

$$s_A = \frac{\vec{r}_A \cdot \vec{v}_A}{r} \equiv \hat{r}_A \cdot \vec{v}_A$$

Instead use the mean difference between the radial velocities of a pair of galaxies:

$$\langle s_1 - s_2 \rangle_p = v_{12}\, \hat{r} \cdot (\hat{r}_1 - \hat{r}_2)/2 \quad \text{where} \quad \vec{r} = \vec{r}_1 - \vec{r}_2$$

To estimate v_{12} we minimize the quantity

$$\chi^2(r) = \sum_{A,B} \left[(s_A - s_B) - p_{AB}\, \tilde{v}_{12}(r)/2 \right]^2$$

$$p_{AB} \equiv \hat{r} \cdot (\hat{r}_A + \hat{r}_B)$$

The sum is over all pairs with some fixed separation r.

Mark III: 2437 spirals and 544 ellipticals

SFI: 1300 late type spirals

ENEAR: 1359 Ellipticals

RFGC: 1327 Spirals

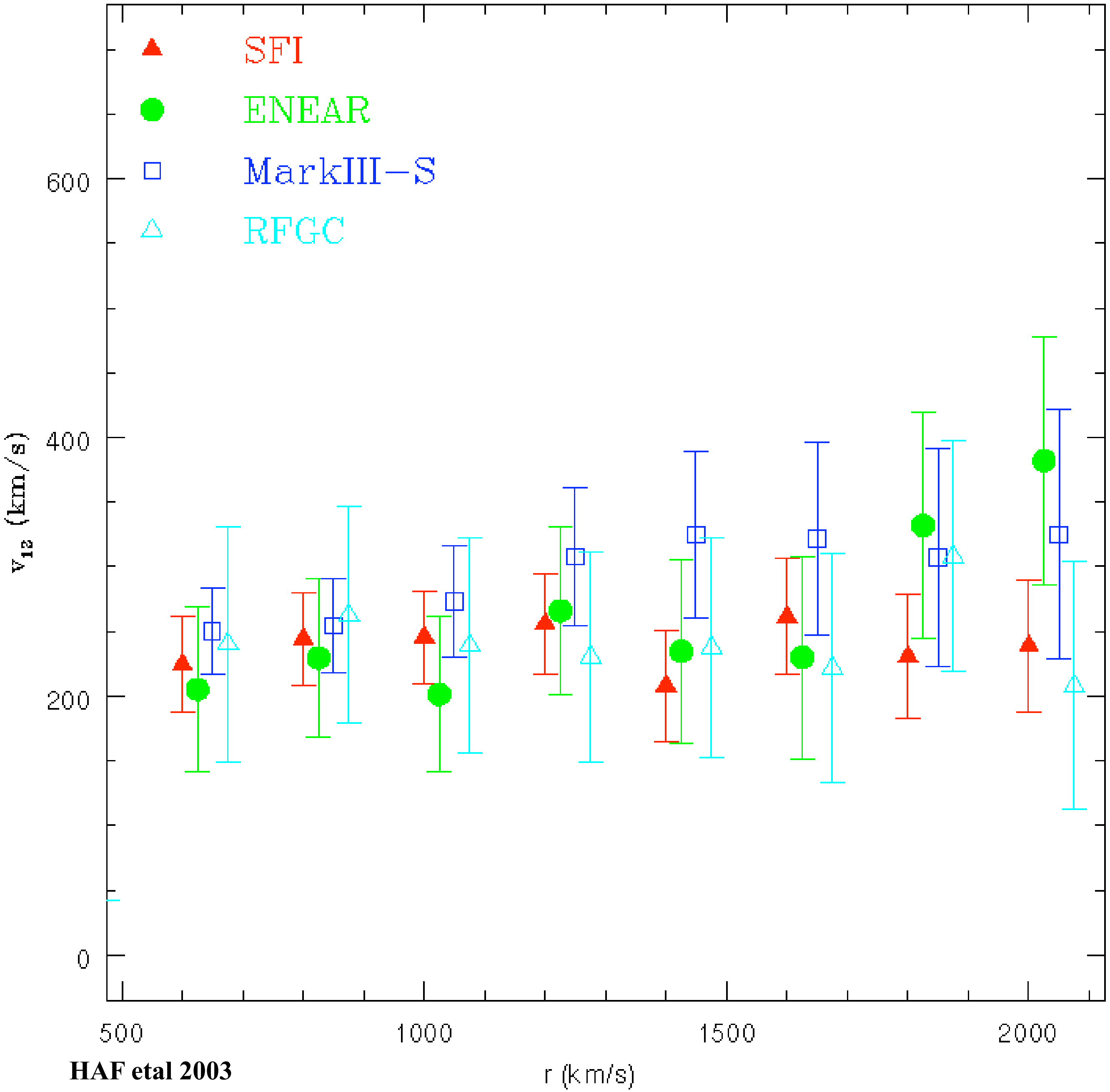
The estimates from the each of the catalogs are similar,

$\Rightarrow$ NO velocity bias

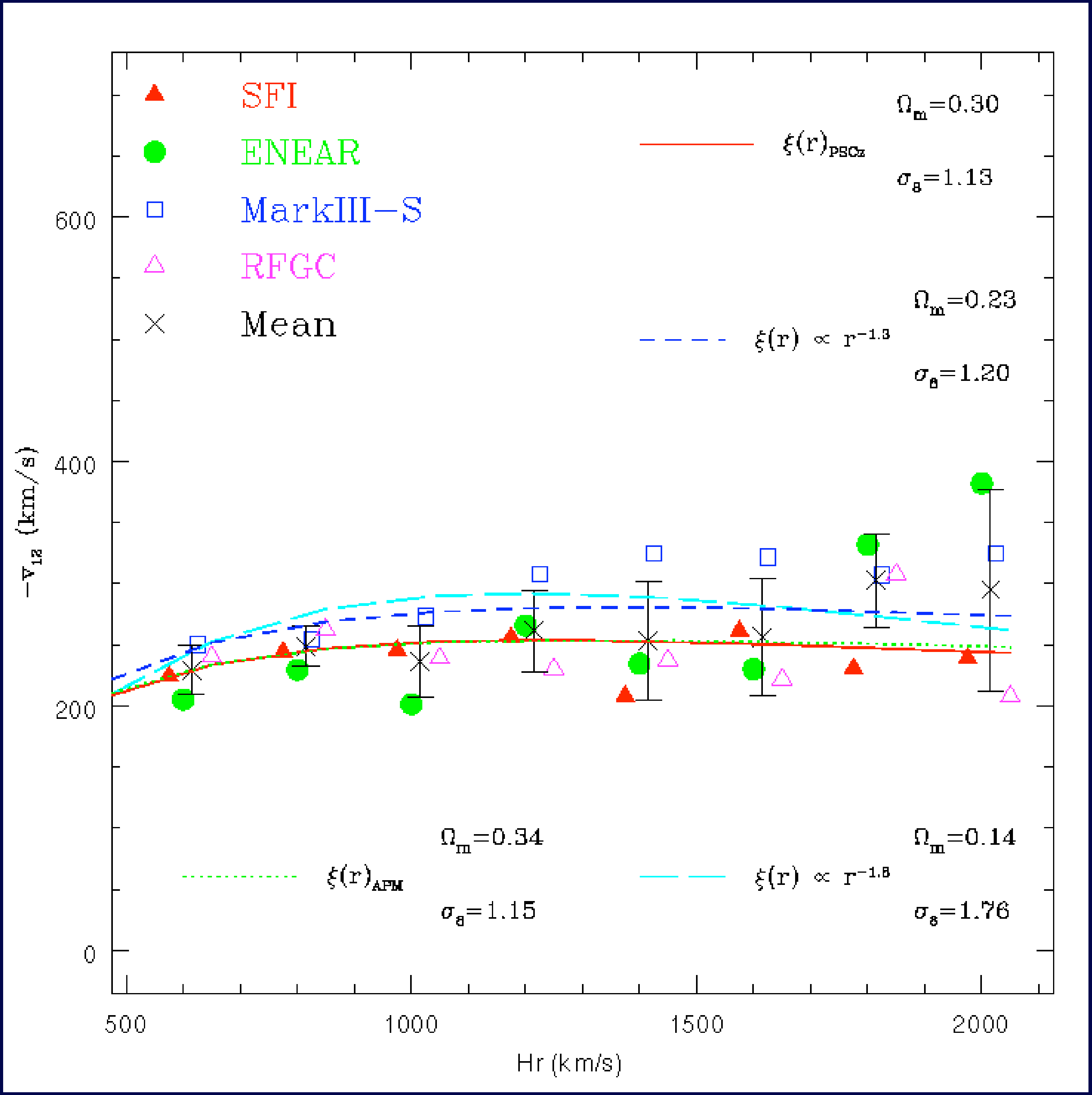
Standard linear bias model assumes: $\delta_g = b\,\delta$

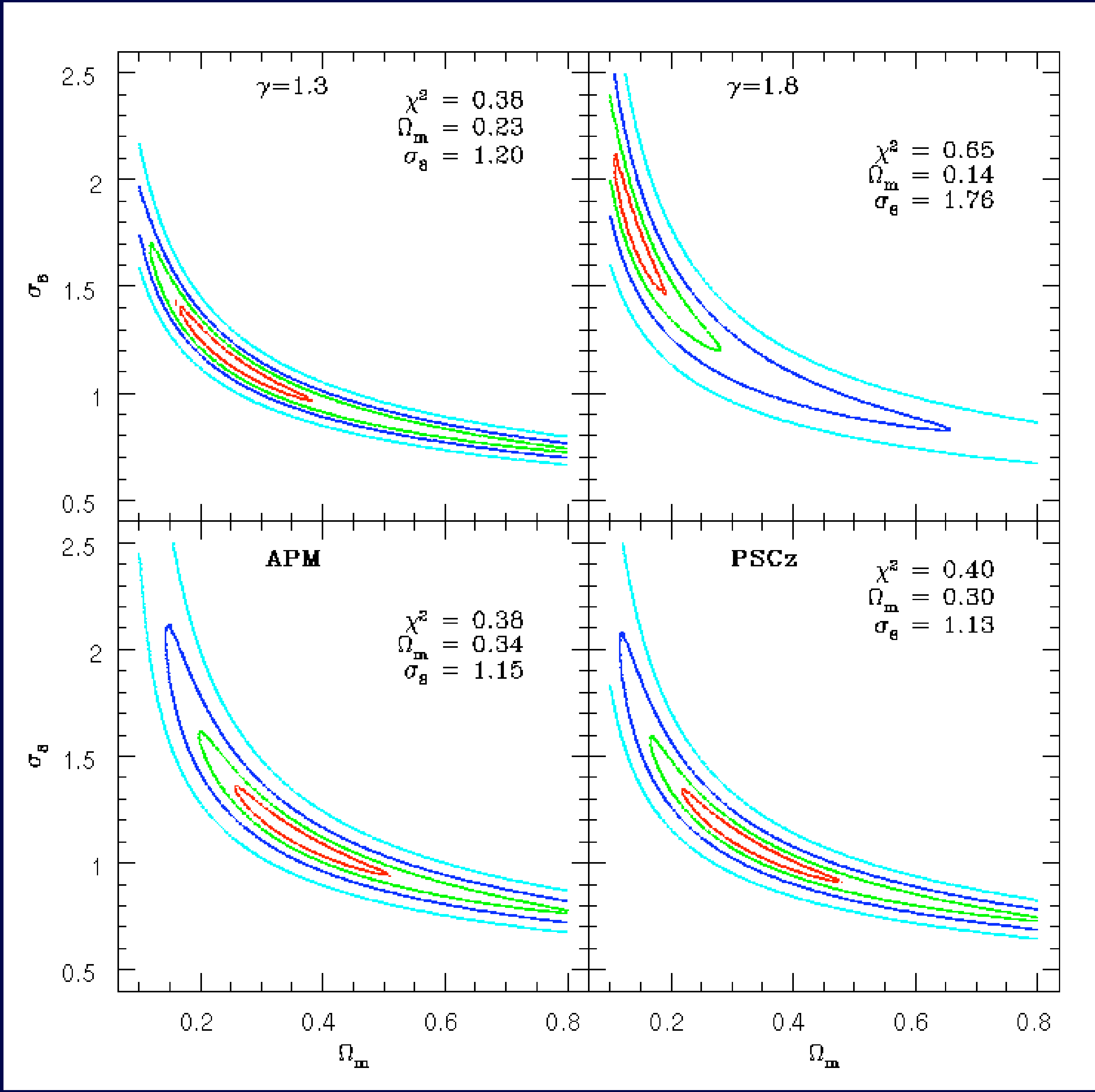
The ratio of v_{12} from the elliptical and spiral samples is:

$$\boxed{\frac{v_{12}^{(E)}}{v_{12}^{(S)}} = \frac{b_E}{b_S}} = 1 \pm 0.15$$



HAF etal 2003


SFI
ENEAR
MarkIII−S
RFGC
Mean
Ω_m=0.30
$\xi(r)_{PSCz}$
σ_8=1.13
Ω_m=0.23
$\xi(r) \propto r^{-1.3}$
σ_8=1.20
Ω_m=0.34
$\xi(r)_{AFM}$
σ_8=1.15
Ω_m=0.14
$\xi(r) \propto r^{-1.8}$
σ_8=1.76
$-v_{12}$ (km/s)
0
200
400
600
500
1000
1500
2000
Hr (km/s)



γ=1.3
χ² = 0.38
Ω_m = 0.23
σ_8 = 1.20
γ=1.8
χ² = 0.65
Ω_m = 0.14
σ_8 = 1.76
APM
χ² = 0.38
Ω_m = 0.34
σ_8 = 1.15
PSCz
χ² = 0.40
Ω_m = 0.30
σ_8 = 1.13
σ_8
Ω_m
0.5
1
1.5
2
2.5
0.2
0.4
0.6
0.8

The VELMOD (Willick etal 1997) analysis constrains

$\beta = \Omega^{0.6}\, \sigma_8$ to be $\beta = 0.5 \pm 0.05$

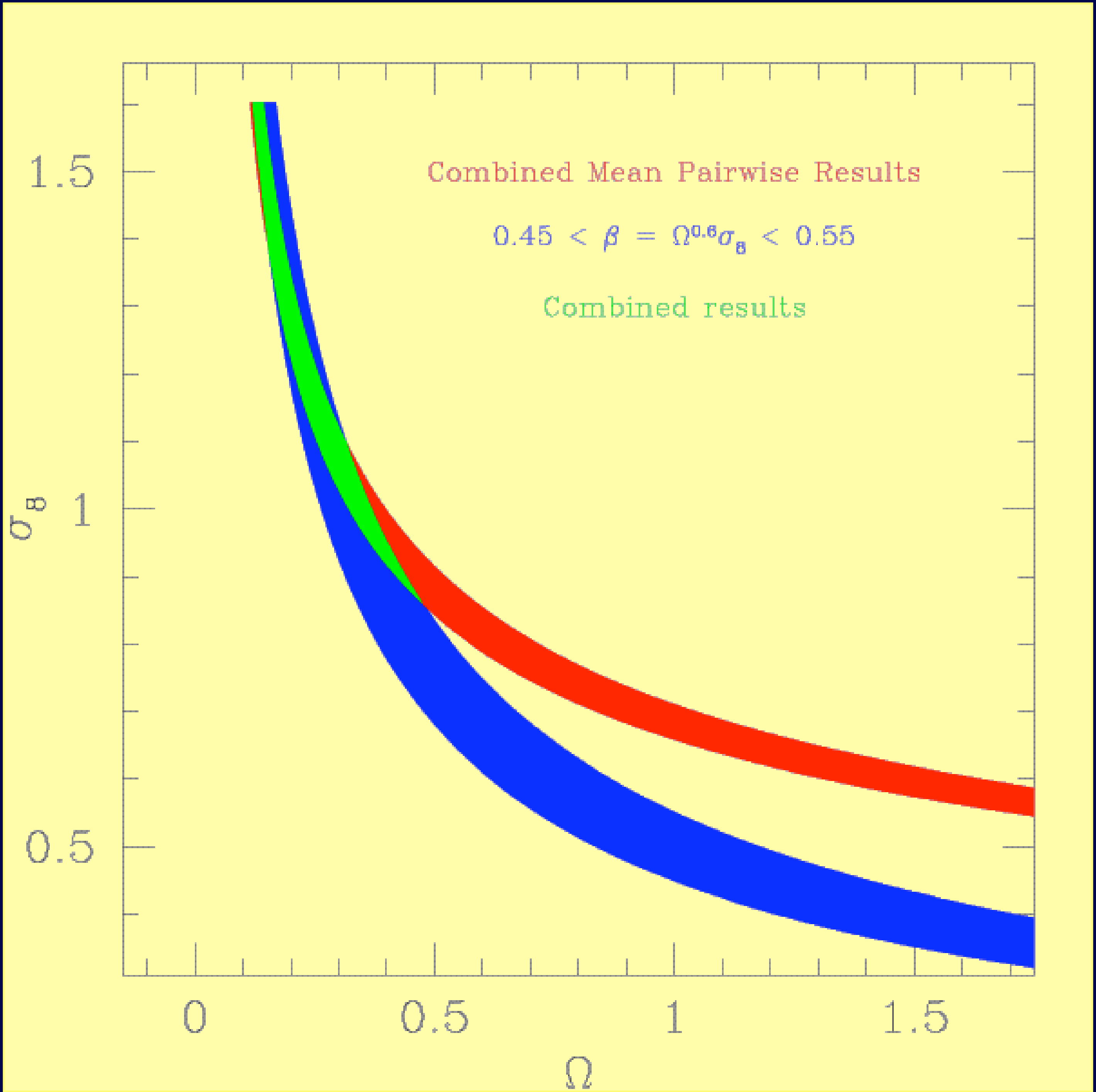
From velocity – density using the Mark III and IRAS 1.2 Jy.

Since the (Ω , σ_8) dependence of v_{12} and β are different we may use both to break the degeneracy between Ω and σ_8.

Results: Low density Universe

$\Omega < 0.5$

$\sigma_8 > 0.9$



Combined Mean Pairwise Results
$0.45 < \beta = \Omega^{0.6}\sigma_8 < 0.55$
Combined results
σ_8
1.5
1
0.5
0
0.5
1
1.5
Ω

Optimal Moments for the Analysis of Peculiar Velocity Surveys

Hume A. Feldman
The University of Kansas
With
Rick Watkins

ApJ 564 534-541 2002

ApJ 599, 820-828 2003.

Removal of non linear effects, aliasing and incomplete cancellations

Karhunen-Loève / Fisher Matrix Formalism

Likelihood Methods for Peculiar Velocities

The analysis of observed line-of-sight velocities:

N objects with positions r_i and line-of-sight velocities v_i.

The observed velocity: $$v_i = \underbrace{\vec{v}(\vec{r}_i)}_{\text{Linear velocity field}} \cdot \hat{r}_i + \delta_i$$ ← noise

The covariance matrix:

$$R_{ij} = < v_i v_j > = R_{ij}^{(v)} + \delta_{ij}(\sigma_i^2 + \sigma_*^2)$$

$$\boxed{\begin{aligned} R_{ij}^{(v)} &= \frac{1}{(2\pi)^3}\int P_{(v)}(k) W_{ij}^2(k) d^3k \\ &= \frac{H^2 f^2(\Omega_0)}{2\pi^2}\int P(k) W_{ij}^2(k) dk \end{aligned}}$$

The probability distribution for the line-of-sight peculiar velocities:

$$L\left(v_1,\cdots,v_N;P(k)\right)=\sqrt{\left|R^{-1}\right|}\exp\left(\frac{-v_i R_{ij}^{-1} v_j}{2}\right)$$

Alternately, given a set of velocities $(v_1,\ldots,v_n)$

$\Rightarrow L\left(v_1,\ldots,v_N;P(k)\right)$ is the likelihood functional for the power spectrum.

Given a power spectrum parameterized by some vector

$$\Theta = (\theta_1,\ldots,\theta_s)$$

$\Rightarrow L\left(v_1,\ldots,v_N;\Theta\right)$ is the likelihood functional for the parameter Θ.

The value of the parameter vector that maximizes the likelihood is Θ_{ML}

Define the Fisher Transformation matrix:

$$\boxed{F_{ij} = \left\langle \frac{\partial^2(-\ln L)}{\partial\theta_i \partial\theta_j} \right\rangle \Bigg|_{\Theta=\Theta_0}}$$

The variances for an unbiased estimators are:

$$\boxed{\Delta(\theta_{ML})_i \geq (F_{ii})^{-1/2}}$$

Cramér-Rao inequality

In the limit of large N this becomes an equality

Here we assume that this limit is satisfied.

Data Compression

Karhunen-Loève methods:
Kendall & Stuart (1969)
Tegmark, Taylor & Havens (1997)
Watkins etal 2002
HAF etal 2003

Replace N original line-of-sight velocities $v_1, \ldots, v_N$ with M moments $u_1, \ldots, u_M$ where $M \leq N$.

Here we concentrate on linear data compression where the moments are:

$$u_i = B_{ij}\, v_j$$

B_{ij} is an $M \times N$ matrix .

If $M < N \Rightarrow$ we lose information.

We arrange it such that the lost information is primarily associated with small scales.

Suppose we compress all the velocity information into a single moment.

$u = b_i\, v_i.$

Where b_i is a $1 \times N$ set of coefficients.

The Fisher matrix for the compressed data is:

$$\boxed{\tilde{F}_{qq} = \frac{1}{2}\left| b_i \frac{\partial R_{ij}}{\partial \theta_q} b_j \right|^2}$$

Where $b_i\, R_{ij}\, b_j = 1$

Since $\Delta\theta_q^{\,2} = 1 / F_{qq}$ we can find a moment that carries the minimum information about θ_q by minimizing the RHS.

Introduce a Lagrange multiplier and extremize with respect to b_i

$$\boxed{\left| b_i \frac{\partial R_{ij}}{\partial \theta_q} b_j \right| - \lambda b_i R_{ij} b_j}$$

Since R_{ij} is symmetric and positive definite we can Cholesky decompose it:

$$R_{ij} = L_{ik} L_{jk}$$

For some invertible Matrix L_{ij}.

Eigenvalue problem:

$$\boxed{\left(L_{ki}^{-1} \frac{\partial R_{ij}}{\partial \theta_q} L_{lj}^{-1} \right)\left(L_{ml} b_m \right) = \lambda \left(L_{jk} b_j \right)}$$

Solving this gives us a set of N orthogonal eigenvectors $L_{ji}(b_n)_j$ with corresponding eigenvalues λ_n.

Eigenvector $L_{ji}(b_n)_j$ $\Leftrightarrow$ $u_n = (b_n)_i v_i$ Moment

The moments u_n are statistically independent, of unit variance:

$\Rightarrow$ Finding λ_n gives us the error bar of θ_q

$\Rightarrow$ If we convert the velocities into N moments there will be no loss of information and the transformation matrix will be invertible.

Since the moments are statistically independent, when we compress the data by removing selected moments, the information contained by those moments will be completely removed from the data.

Moment Selection

Order the moments in order of increasing eigenvalues:

$$\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$$

Each moment caries successively more information about θ_q with u_n caries the maximum possible amount of information.

Goal:

Produce a data set that is less sensitive to the value of θ_q and keep as many moments as possible to retain the information about large scales.

Criterion:

1) Estimate $\theta_q = \theta_{qo}$

2) Keep the largest number M such that $\Delta\theta_q \geq \theta_{qo}$.

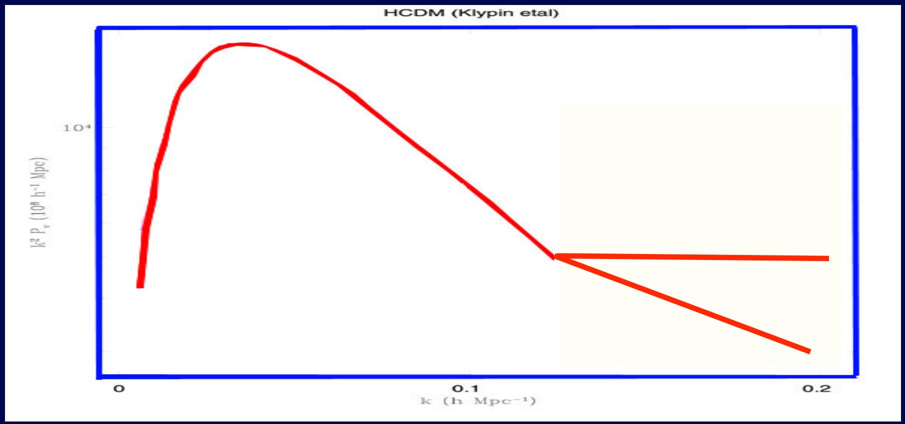
If our estimate of θ_q is correct

The set of moments $u_1,\ldots,u_M$ will not contain enough information to distinguish θ_q from zero.

Power Spectrum Model

Assume that:

$$P(k) = P_l(k) + \theta_q P_{nl}(k)$$



Where $P_l(k) = 0$ for $k > k_{nl}$

$P_{nl}(k) = 0$ for $k < k_{nl}$

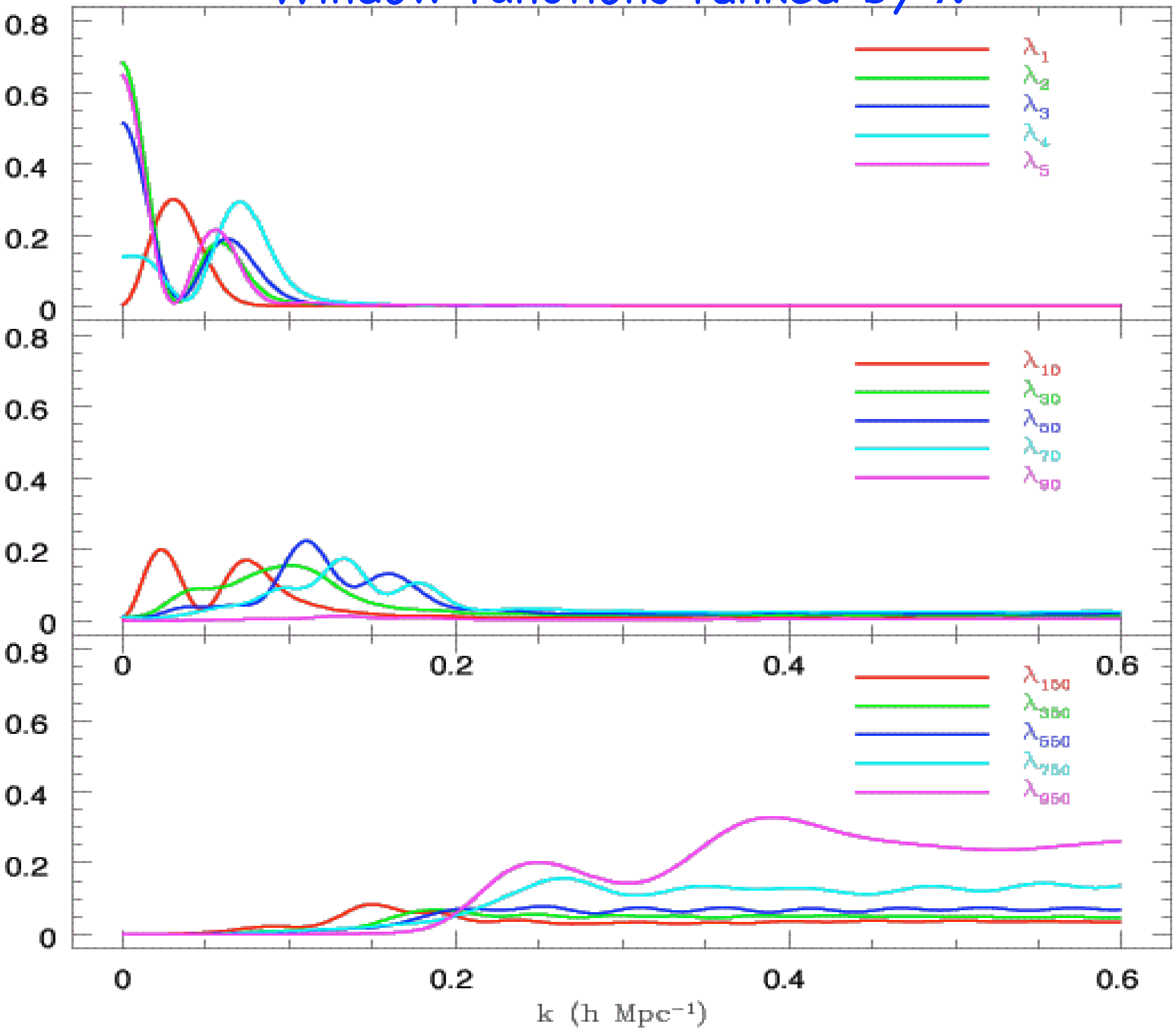
Use the BBKS power spectrum for $P_l(k)$.

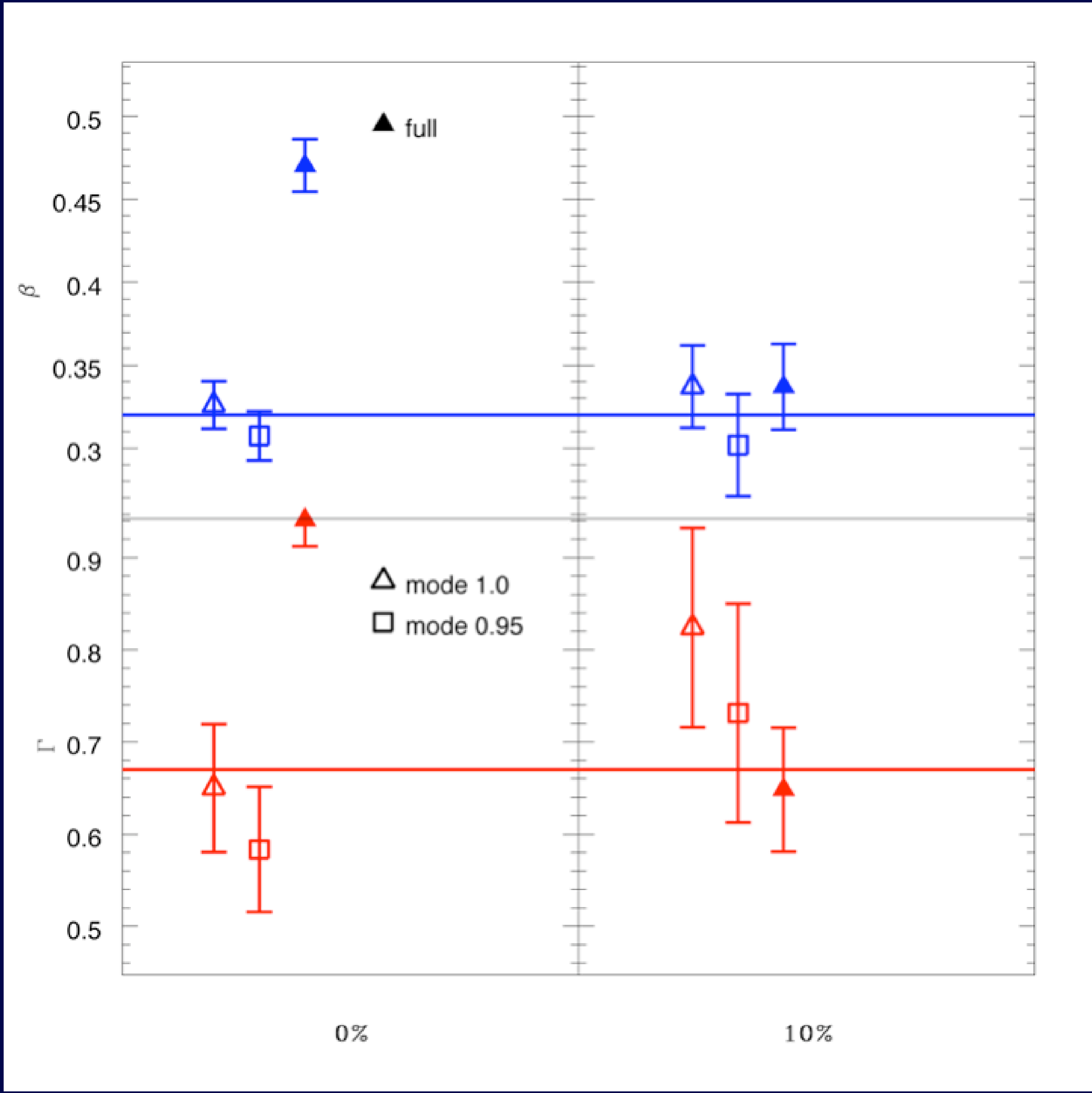
Try e.g. $P_{nl}(k) \propto k^{-1}$

We choose $P_{nl}(k) = P_0$ for $k_{nl} < k < k_c$.

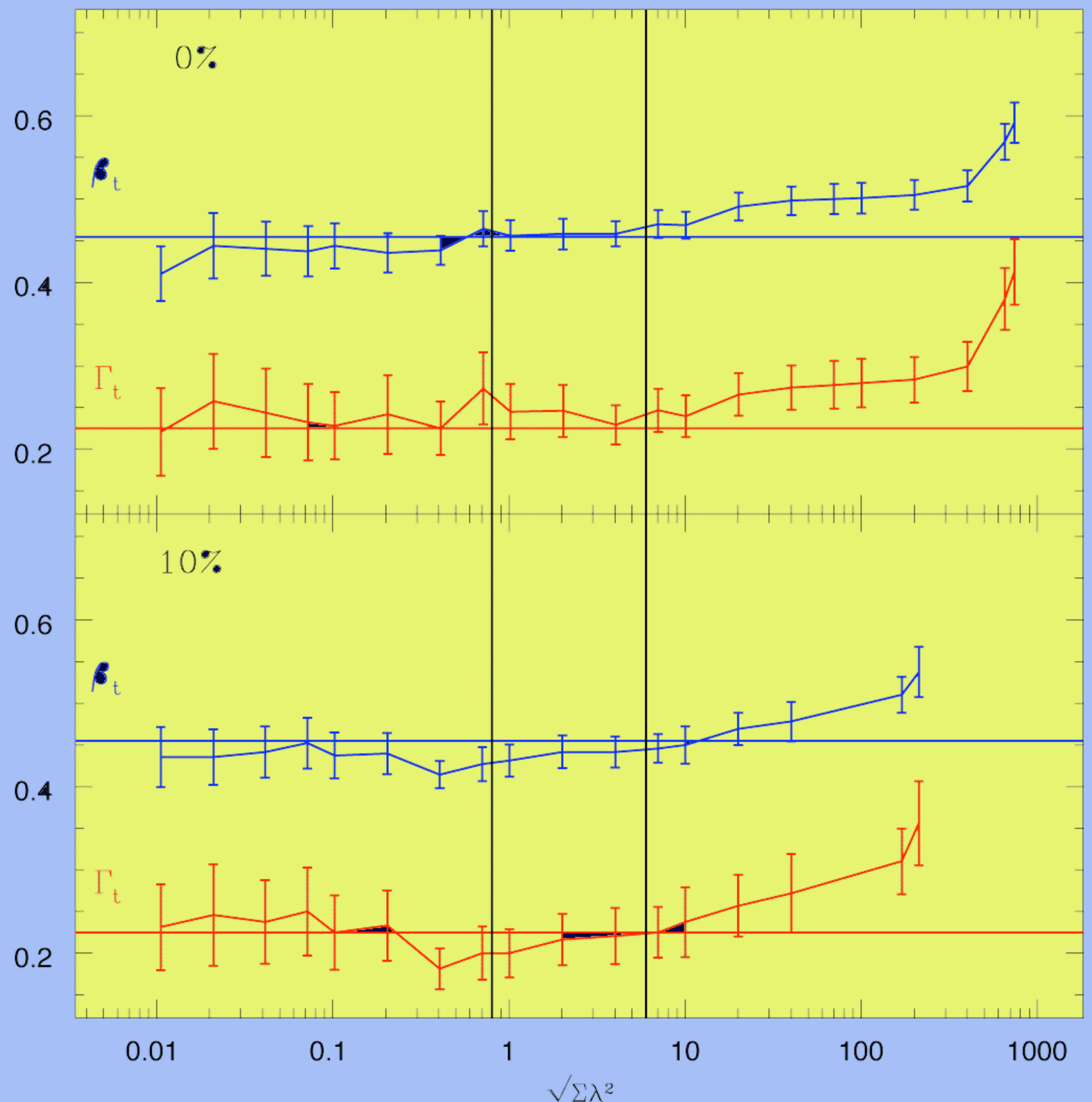
Where contribution of nonlinear scales to line-of-sight velocity dispersion ($\sigma*$) should equal the estimate from the data.

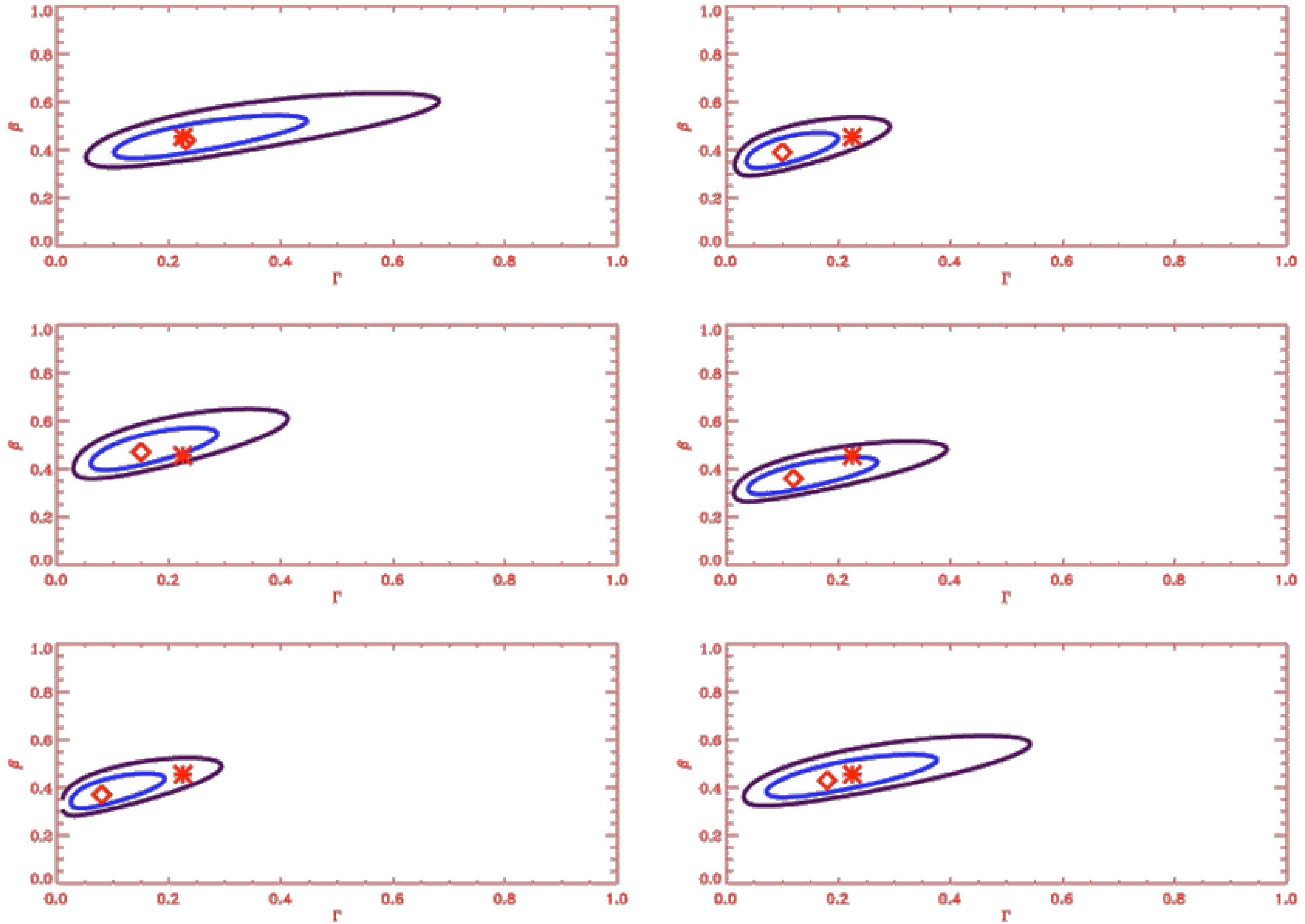
Window functions ranked by λ



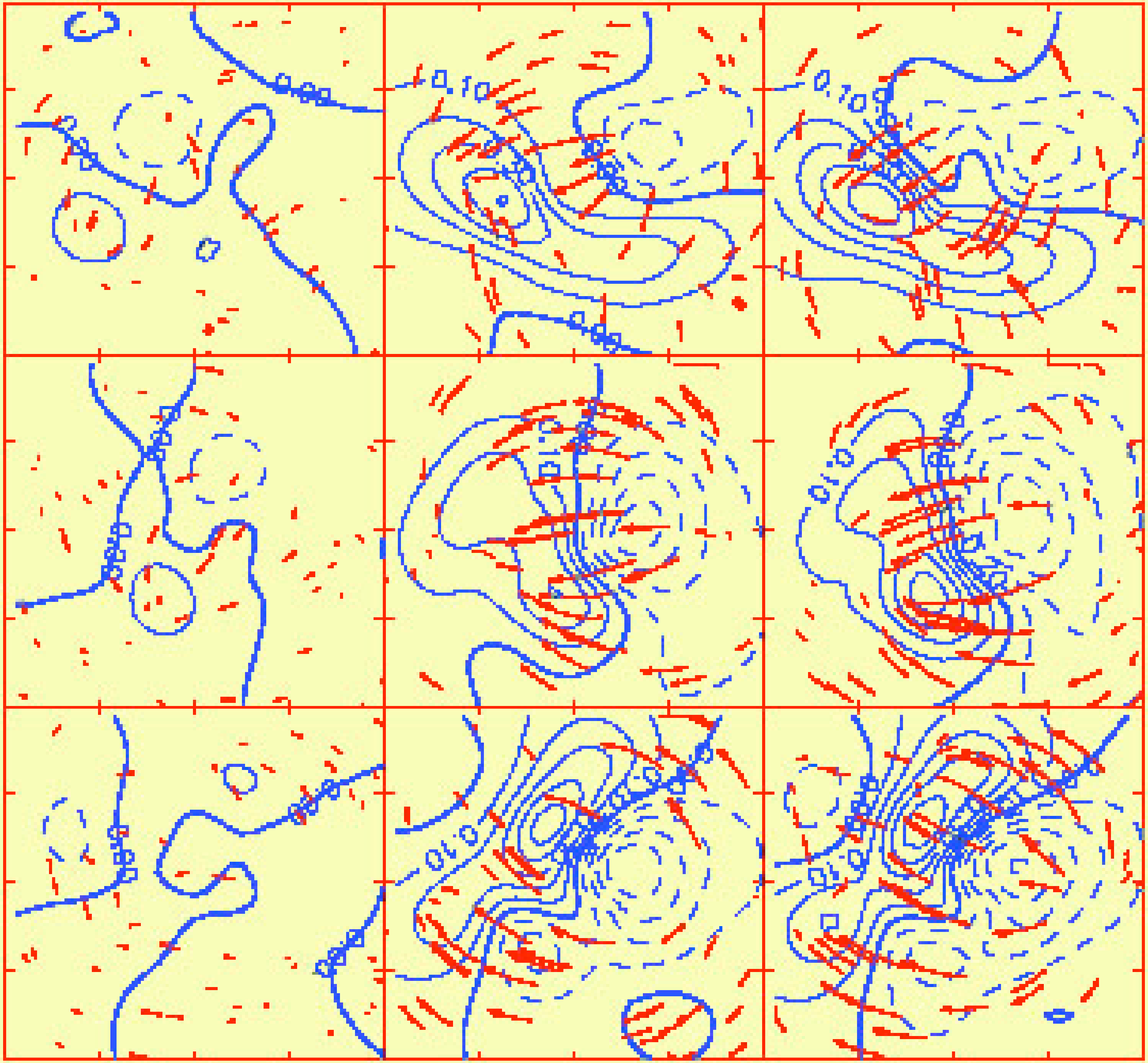


0.5
0.45
0.4
0.35
0.3
β
full
0.9
0.8
0.7
0.6
0.5
Γ
mode 1.0
mode 0.95
0%
10%


0%
0.6
0.4
0.2
β_t
Γ_t
10%
0.6
0.4
0.2
β_t
Γ_t
0.01
0.1
1
10
100
1000
$\sqrt{\Sigma\lambda^2}$



1.0
0.8
0.6
0.4
0.2
0.0
β
0.0
0.2
0.4
0.6
0.8
1.0
Γ

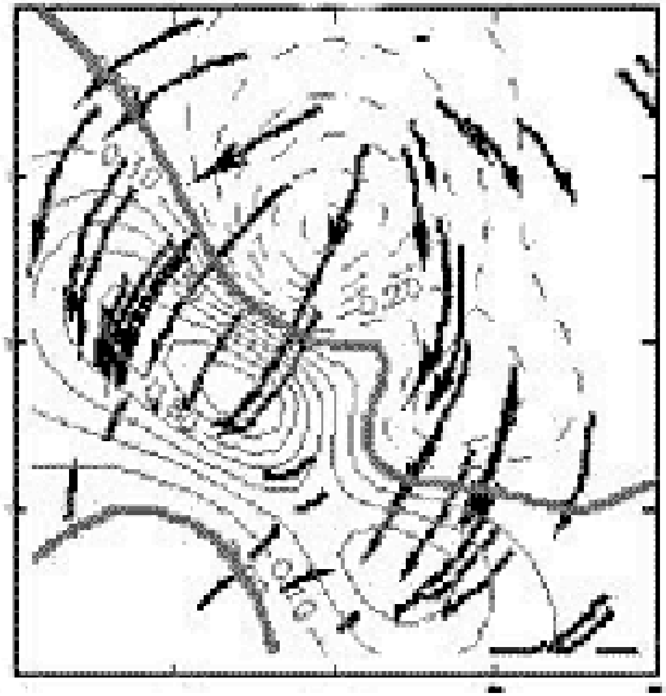


xy–plane

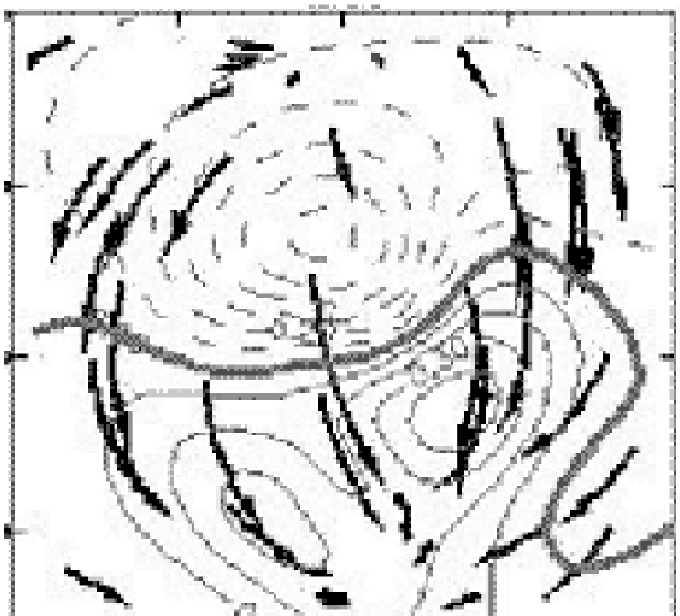
HAF, Hoffman & Nusser

xz–plane

yz–plane



High density Regions



Low density Regions

Cosmological Constant with velocity Fields

$$\left(\frac{H}{H_o}\right)^2 = \Omega_o(1+z)^3 - \Big(\Omega_o + \Lambda_o - 1\Big)(1+z)^2 + \Lambda_o$$

$$\Omega(\Omega_o, \Lambda_o, z) = \Omega_o\left(\frac{H}{H_o}\right)^{-2}(1+z)^3$$

$$f(\Omega_o, \Lambda_o, z) = X^{-1}\Big(\Lambda_o(1+z)^{-2} - \frac{1}{2}\Omega_o(1+z)\Big) - 1$$
$$+(1+z)^{-1}X^{-3/2}\left(\int_0^{(1+z)^{-1}} X^{-3/2}da\right)^{-1}$$

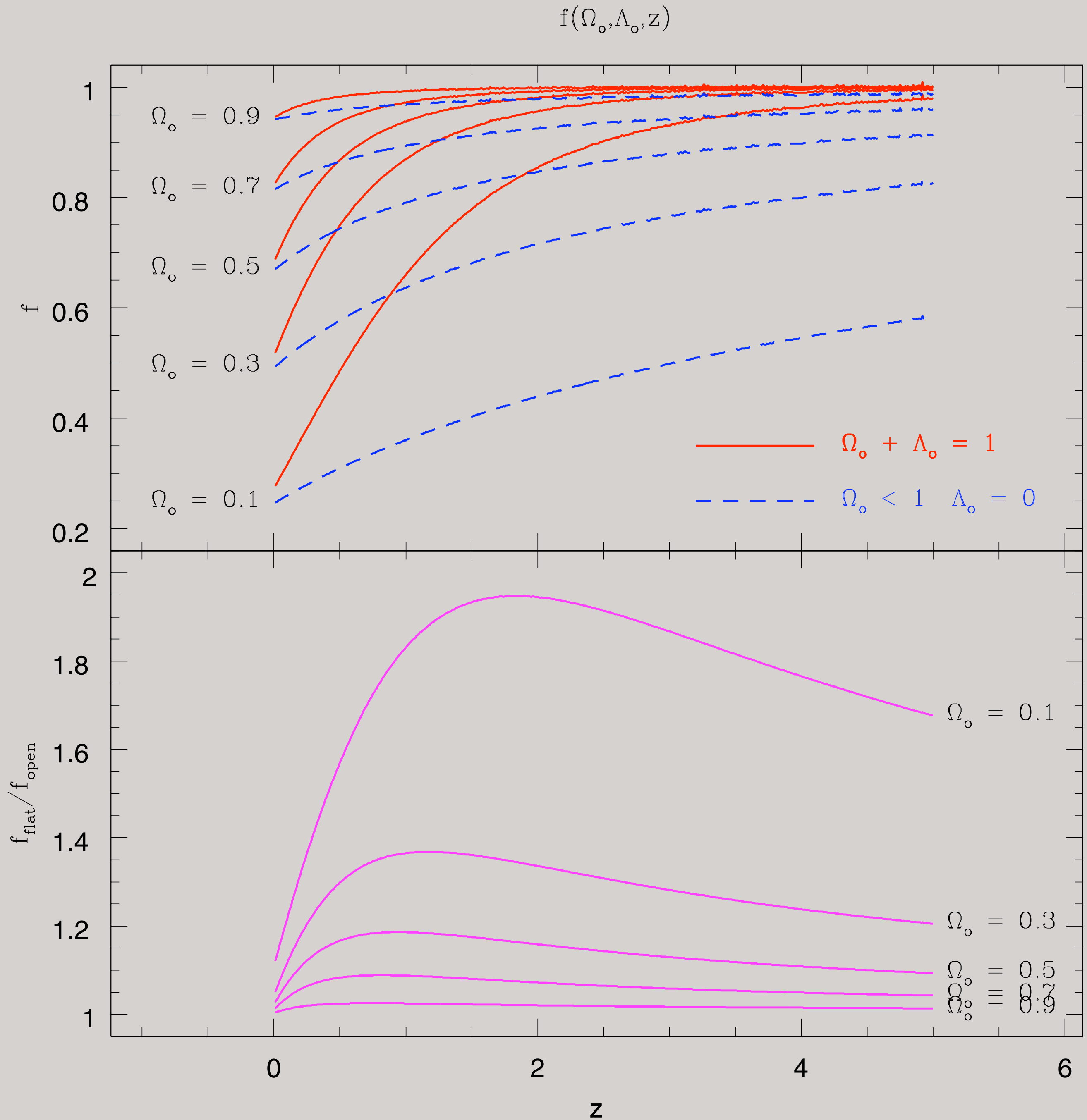
Where

$$X = 1 + \Omega_o z + \Lambda * (1+z)^{-2} - 1$$

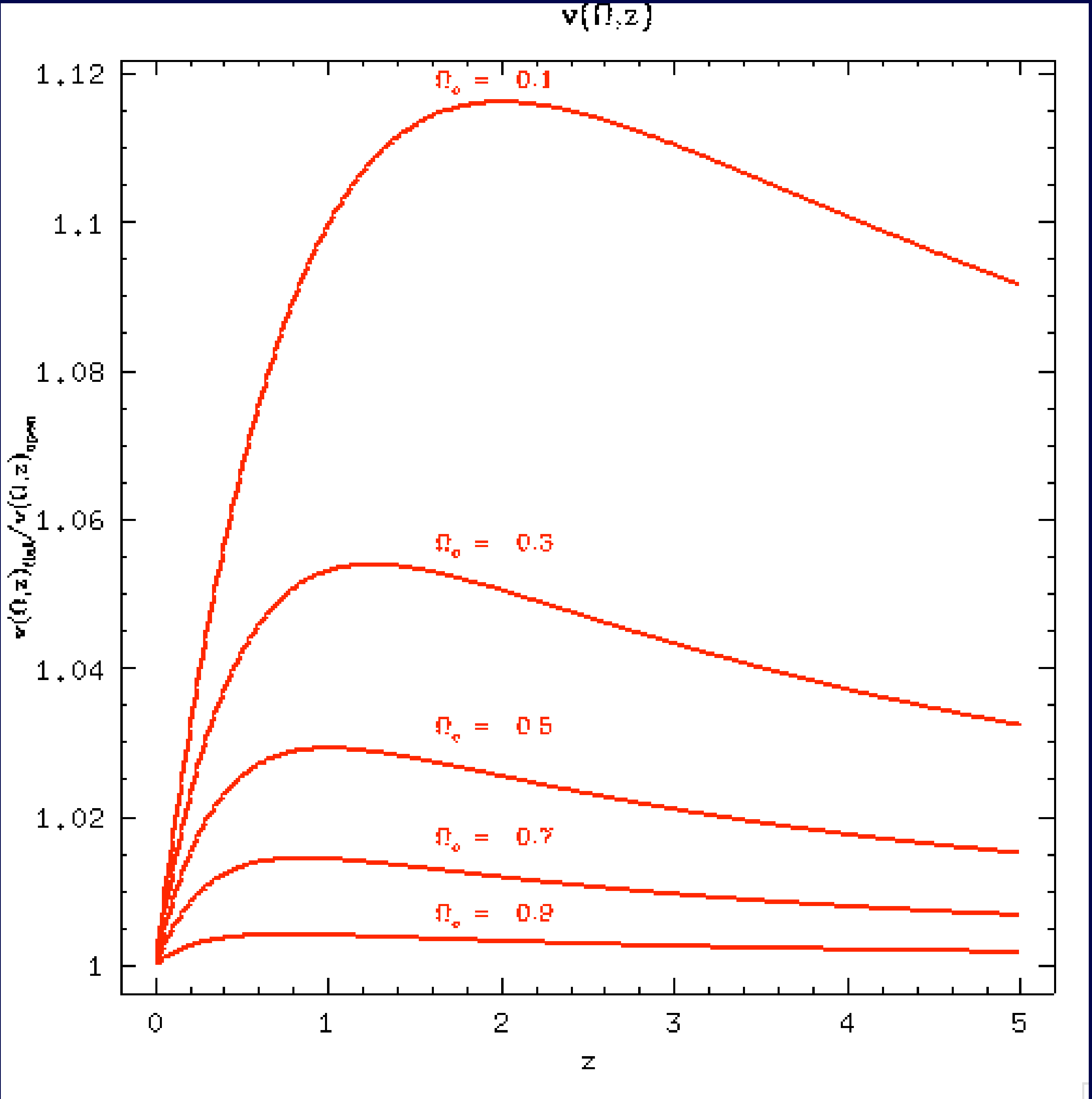
$$\frac{H_{flat}}{H_{open}} = \left(\frac{1 + \frac{(\Omega_o^{-1} - 1)}{1+z}}{1 + \frac{(\Omega_o^{-1} - 1)}{(1+z)^3}} \right)^{-0.5}$$

$$\frac{f_{flat}}{f_{open}} = \left(\frac{1 + \frac{(\Omega_o^{-1} - 1)}{1+z}}{1 + \frac{(\Omega_o^{-1} - 1)}{(1+z)^3}} \right)^{0.6}$$

$$\frac{v_{flat}}{v_{open}} = \left(\frac{1 + \frac{(\Omega_o^{-1} - 1)}{1+z}}{1 + \frac{(\Omega_o^{-1} - 1)}{(1+z)^3}} \right)^{0.1}$$



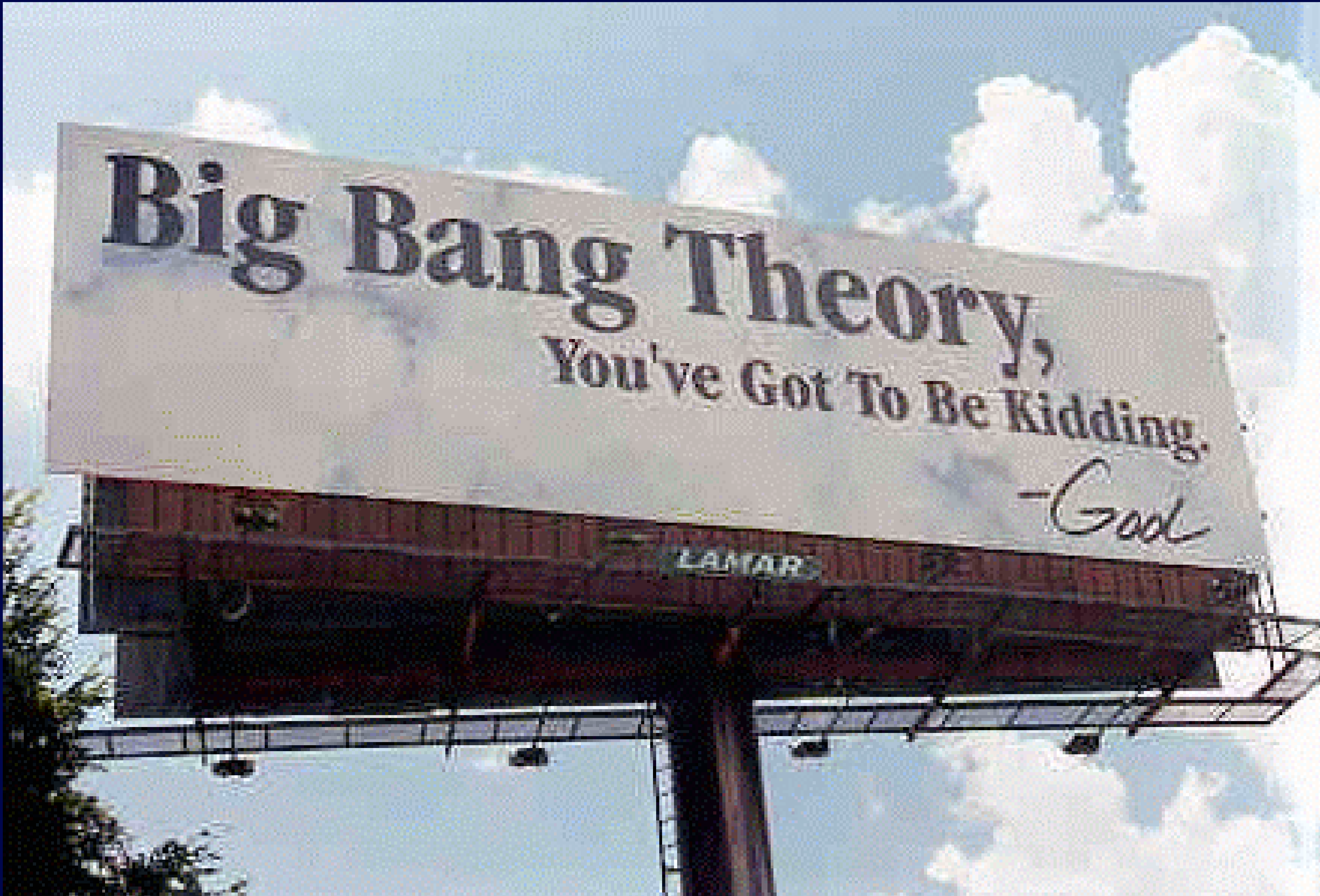
$f(\Omega_o,\Lambda_o,z)$
$\Omega_o = 0.9$
$\Omega_o = 0.7$
$\Omega_o = 0.5$
$\Omega_o = 0.3$
$\Omega_o = 0.1$
$\Omega_o + \Lambda_o = 1$
$\Omega_o < 1 \quad \Lambda_o = 0$
f
f_{flat}/f_{open}
$\Omega_o = 0.1$
$\Omega_o = 0.3$
$\Omega_o = 0.5$
$\Omega_o = 0.7$
$\Omega_o = 0.9$
z


v(Ω,z)
1.12
1.1
1.08
1.06
1.04
1.02
1
0
1
2
3
4
5
z
$v(\Omega,z)_{flat}/v(\Omega,z)_{open}$
Ω_o = 0.1
Ω_o = 0.3
Ω_o = 0.5
Ω_o = 0.7
Ω_o = 0.9

Concluding remarks

- Distance measurements are good.
- Determination of H_o, Ω, Λ, σ_8, b, …
- Mapping of the Large-scale peculiar velocity field
- Study of the gravitational potential
- A 'true' tracer of the mass distribution
- We have some robust and consistent statistics
- We need:
- Deeper and denser surveys
- More accurate distance measurements

Velocity Fields Forever


Big Bang Theory,
You've Got To Be Kidding.
-God
LAMAR